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MODELLING THE TREE GROWTH IN MIXED TROPICAL FORESTS I. USE OF DIAMETER AND BASAL AREA INCREMENTS

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WAN RAZALI MOHD. 1988. Modelling the tree growth in mixed tropical forests I. Use of diameter and basal area increments. Tree growth can be expressed by either diameter increment (dD) or basal area increment (dBA). Little work has been done to determine which of these two parameters is the appropriate dependent variable for use in growth models. This paper examines the growth of regenerated mixed tropical forests in Peninsular Malaysia which were measured over a period of 13-20 *y*. Least squares regression equations were developed to relate dD and dBA to initial tree diameter at breast height (DBHOB) in 36 permanent growth plots of 0.4 *ha* each. The residual plots relating dD and dBA to DBHOB show the existence of non-homogeneity of variance. The transformations of dD and dBA to remove non-homogeneity of variance were carried out and used to relate to DBHOB. Furnival Index was then constructed and adjusted to compare likelihoods of different statistical models for dependent variables that have been expressed in the same sample space. The result indicates that diameter increment is a more appropriate dependent variable to be used in growth models in mixed tropical forests.

Key words: Growth modelling - mixed tropical forests - index-of-fit - power transformation - likelihood functions.

Introduction

The growth of individual trees may be expressed by either diameter increment or basal area increment. If we consider two trees with the same diameter increment but different initial diameters, then these two trees would have different basal area increments. The above consideration poses an initial problem in deciding the appropriate form of the dependent variable (diameter increment or basal area increment) because it may be felt intuitively that these two variables expressed increment (or growth) somewhat differently.

For many researchers, the decision to use diameter or basal area increment in growth studies seems to have been decided arbitrarily. Lemmon and Schumacher (1962), Newnham (1966), Rudra and Filmer (1970), Goulding (1972), Hegyi (1973), Lanford and Cunia (1977) and Hahn and Leary (1979) used diameter increment while others, such as Opie (1968, 1972) and Moore *et al.* (1973) used basal area increment in their growth studies. None of these workers gave any reasons for preferring one parameter over the other.

On the other hand, Cole and Stage (1972) and Stage (1973) analysed radial increment, basal area increment and the logarithms of each. They found that, using Furnival's (1961) index of fit criterion, the logarithm of basal area increment to be the superior form of the dependent variable. Hann (1980) decided to use the natural logarithm of basal area and then converted that value to a diameter growth rate. He, like Cole and Stage (1972) and Stage (1973), considered basal area growth as nearly linear over short time periods and the residuals of the logarithm of basal area growth more often approached normality with homogeneous variance as basic reasons for his choice of that variable. West (1980) found that the precision of estimate of future diameter is the same whether diameter or basal area increment is used. He concluded that no *a priori* reason exists for expressing growth as diameter or basal area increment. Manley (1981), using Furnival's index of fit criterion, found diameter increment to be better as the dependent variable.

Based on the above review, it can be seen that the question of whether to use diameter or basal area increment in growth studies has not been settled definitely. Such a question is even harder to answer in growth studies of mixed tropical forests where such growth analyses have never been done. Nevertheless, I decided to examine the prediction of future diameter of individual trees from regression relationship using both diameter and basal area increments as dependent variables despite a lack of growth studies in mixed tropical forests

Source of data and regression methods

Data from permanent growth plots (0.4 *ha* each), measured over a growth period of 13-20 *y* in the regenerated forests of Peninsular Malaysia (Wyatt-Smith 1963), were used in this study. The forests are dominated by the family Dipterocarpaceae, rich in red meranti (one group of *Shorea* spp.) and keruing (*Dipterocarpus* spp.). The average diameter (DBH) for Dipterocarps (DIPT) was 37.6 *cm*, and for Non-Dipterocarps: Light Hardwood (LHW) 29.2 *cm*, Medium Hardwood (MHW) 28.9 *cm*, and Heavy Hardwood (HHW) 24.5 *cm*. The average basal area of trees 10 *cm* dbh and above at the time of plot establishment was 16.9 m^2 ha^{-1}.

A total of 54 growth plots were established as early as 1962 and remeasured at intervals of 3 - 9 *y*. For the present work, 36 plots were randomly selected and used in the analyses. Diameter increment (dD), basal area increment (dBA) and initial tree diameter at breast height (DBHOB) of the four species groups selected were subjected to the regression analyses. The species groups are DIPT, LHW, MHW, and HHW. The number of diameter measurements varied between plots but ranged between three to six. The number of observations available for each species group is shown in Table 1. Background information of these 54 growth plots are presented in Wan Razali (1986).

Table 1. Species group composition, number of trees, and total number of tree measurement periods used in growth analysis

Species group (1)	Number* of trees measured (2)	Number of tree** measurement periods (3)	Number in (3) used in growth analysis (4)
DIPTEROCARPS:	1542	3631	2576
NON-DIPTEROCARPS:			
Light Hardwood	3178	6046	4076
Medium Hardwood	3374	6065	4168
Heavy Hardwood	2746	4660	3203

*Includes trees with one measurement period.
**Excludes trees with one measurement and trees having diameter increment >3 *cm/y*.

For each species group, examination of graphs of dD and dBA plotted against DBHOB suggest a nonlinear relationship. Therefore, for each species group and using data from all increment periods of the 36 growth plots, two regression equations were initially investigated: one equation related dD ($cm\,y^{-1}$) to DBHOB, and the other related dBA ($cm^2\,y^{-1}$) to DBHOB.

Results and discussion

Relationship between initial tree diameter and increment

For relationships relating diameter and basal area increments to DBHOB, examinations of residual plots (Figure 1) suggest the existence of non-homogeneity of variance. There is also an abrupt edge along the line $y = -x$ which corresponds to the negative and zero observed diameter or basal area growth. The trend of non-homogeneity was checked across the intervals of diameter, the predictor variable in the regression. The variance of the residuals of the fitted regression seems to be proportional to the square of initial diameter.

In order to stabilize the non-constant variance, logarithmic transformations were carried out on the dependent variable of diameter and basal area increments. A problem arose with the logarithmic transformation of increment data because of the presence of zero and negative increments. As such, all negative increments were assumed as zero and the actual transformation used was in the form of Ln(Y+1). Weighted least squares regressions were also carried out.

The logarithmic transformation did not appear to improve the residual distribution to a large extent for both diameter increment and basal area increment (Figure 2). Although much of the heteroscedasticity disappeared, there is still a linear trend in residual distribution, especially in the basal area increment. The weighted least squares relating diameter increment to DBHOB appear to deviate slightly from the norm as judged from the residual plot, but the residual plot relating basal area increment to DBHOB still shows a linear trend (Figure 3).

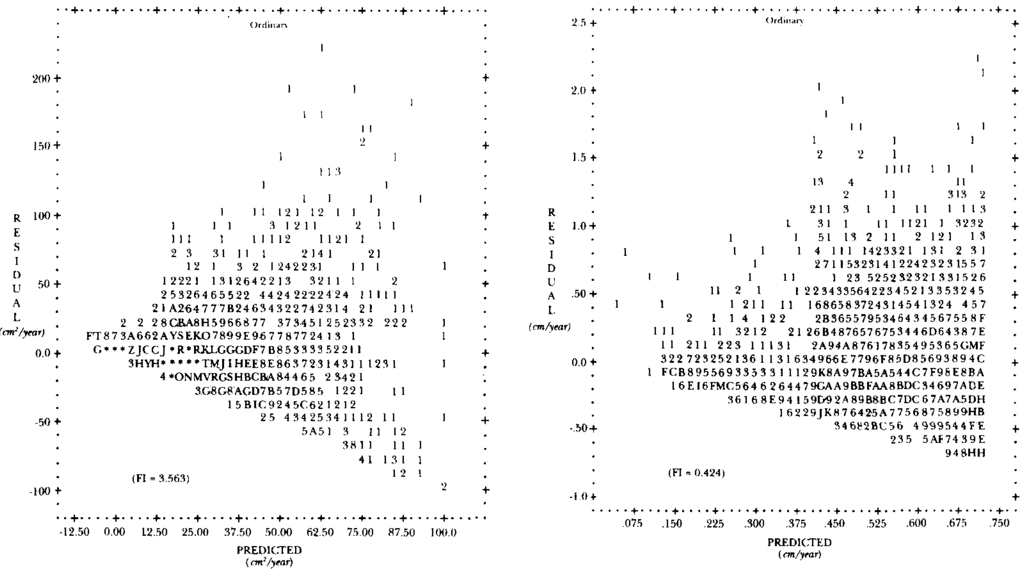

Figure 1. Residuals from ordinary least squares regression plotted against predicted values. The fitted regressions were as follows: $dD = b_0 + b_1D + b_2D^2$ and $dBA = b_0 + b_1D + b_2D^2$ where dD = diameter increment (*cm* y^{-1}), dBA = basal area increment (*cm*^{2} y^{-1}), and D =DBHOB. The result is shown for Dipterocarps; other species showed similar effect. (1, 2, 3,...... 9, A, B, C, ...Z denote the number of points plotted)

Index of fit

Now, the problem is to determine the most appropriate form of the dependent variable. Furnival's (1961) "index of fit" (FI) criterion was used for this purpose. Furnival's index of fit employs the concept of maximum likelihoods and has the advantage of reflecting both the size of residuals and possible departures from linearity, normality and homoscedasticity.

The Furnival index is expressed as

$$FI = [f'(Y)]^{-1} s$$

where $[f'(Y)]^{-1}$ is geometric mean of the derivative of the dependent variable with respect to diameter increment (dD) or basal area increment (dBA) as in the case of this study, and s is the standard error of the fitted regression.

The concept employed by Furnival (1961) is equivalent to transforming (power transformation) the response Y as suggested by Box and Cox (1964). The transformation of Y is equivalent to fitting the model

$$\lambda = X\beta + e, \ \mathrm{var}(e) = \sigma^2 I,$$

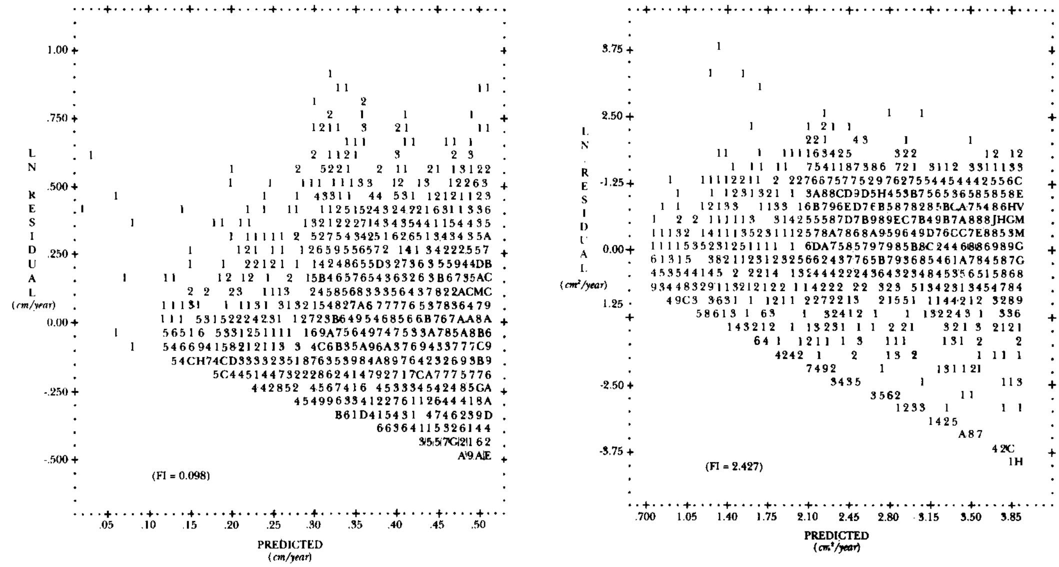

Figure 2. Residuals from logarithmic regression plotted against predicted values. Models fitted were as follow:
$Ln(dD) = b_0 + b_1D + b_2D^2$ and $Ln(dBA) = b_0 + b_1D + b_2D^2$ where dD = diameter increment (*cm/year*), dBA = basal area increment (*cm²/year*), and D = DBHOB. The result is shown for Dipterocarps; other species showed similar effect.
(1, 2, 3, ..., 9, A, B, C, ...,Z denote the number of points plotted)

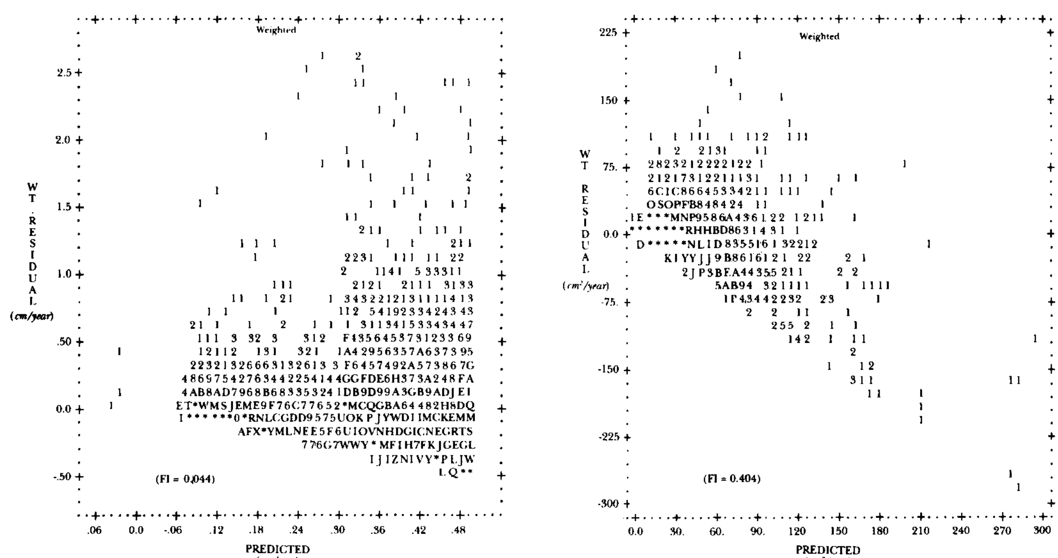

Figure 3. Residuals from weighted least squares regression plotted against predicted values. The weighted regressions fitted were as follows:
$dD = b_0 + b_1D + b_2D^2$ and $dBA = b_0 + b_1D + b_2D^2$ where dD = diameter increment (*cm/year*), dBA = basal area increment (*cm²/year*), and D = DBHOB. The result is shown for Dipterocarps: other species showed similar effect.
(1, 2, 3, ..., 9, A, B, C, ...Z denote the number of points plotted)

where λ is the power of the transformation. If, by convention, we take $\lambda = 0$ to be the natural or log transformation, then all the usual transformations are included: $\lambda = 1$ corresponds to no transformation, and $\lambda = -1$ corresponds to reciprocal. Box and Cox (1964) have suggested that an estimate of λ can be obtained by finding the joint maximum likelihood estimates of B, σ^2, λ and using distributional assumptions concerning **e**. If one assumes that $e \sim N(0, I\sigma^2)$, Weisberg (1980) outlined a procedure by which a usual regression program can be used to obtain the maximum likelihood estimates of λ, B, and σ^2.

FI (or Box and Cox method) was constructed to compare likelihoods of different statistical models for a single response variable. Example, to determine whether dD or Ln(dD) or (dD/DBH^2) better satisfies a linear regression model with normally distributed error with constant variance. FI was not derived to compare regression models for intrinsically different responses such as dD and dBA. This is because likelihood functions of the dependent variables, required in calculating FI, have to be expressed in the same sample space (Furnival 1961). However, dD and dBA represent two different sample spaces, and FI for these responses represents different units of measurement.

The FI reduces to the usual estimate of the standard error when the dependent variable is dD or dBA as in this case. When the dependent variable is some function of dD or dBA, the index may be regarded as an average standard error transformed to units of dD or dBA. Clearly, these two units are different.

To overcome this problem, the derivative of the dependent variable dBA (f'(dBA)) may be expressed with respect to its square root ($\sqrt{dBA}$). This allows both derivatives, f'(dD) and f'(dBA), to be expressed with respect to one common unit ($cm\ y^{-1}$) of the dependent variables dD and dBA (see formula, Table 2). Then, a direct comparison of the index between the two dependent variables may be made for descriptive purposes.

Table 2. Furnival's Index of Fit (FI)

Dependent Variable	Species Groups			
	Dipterocarps		Non-Dipterocarps	
	DIPT	LHW	MHW	HHW
Unweighted:				
Basal Area Increment (dBA)	3.563[a]	3.969	4.389	3.696
Logarithmic:				
Ln(dD)	0.098	0.051	0.052	0.032
Ln(dBA)	2.427[b]	1.584	1.544	1.119
Weighted:				
Diameter Increment (dD)	0.044	0.025	0.029	0.033
Basal Area Increment (dBA)	0.404[c]	0.345	0.323	0.299

Models fitted were: $f(X) = b_0 + b_1D + b_2D^2$, where f(X) is either dD($cm\ y^{-1}$), dBA($cm^2\ y^{-1}$), Ln(dD), or Ln(dBA), and D is DBHOB. The best fit model for each species group is indicated by the lowest FI value.
[a] $FI = [2\sqrt{dBA}]^{-1} * s$ — unweighted; [b] $FI = [2/\sqrt{dBA}]^{-1} * s$ — Logarithmic; [c] $FI = [2\ dBA\ \sqrt{D^2}]^{-1} * s$ — weighted by ($1/D^2$); (The LHW, MHW, and HHW follow the same formula).

Values of FI are given in Table 2. The index of diameter increment indicates better fit than its logarithm and superior than the basal area increment and its logarithm. The normally distributed residuals of diameter increment (Figure 3) further support this conclusion. Thus, diameter increment was chosen as the most appropriate dependent variable for subsequent analyses.

Conclusion

One of the objectives in most modelling studies is to predict the future growth of trees resulting from normal growth or the application of some treatment. It has been shown that, in mixed tropical forests particularly in Peninsular Malaysia, the use of diameter increment gives better fit than its logarithm, and superior than the basal area increment.

It means that either diameter increment or basal area increment is suitable in examining growth effects in these forests. This would then serve as a guide to tree growth modelers as to the most appropriate choice of dependent variable for subsequent analysis.

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