AN APPLICATION OF MATRIX MODELLING TO GROWTH PREDICTION IN SOLOMON ISLANDS RAIN FOREST

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TENNENT, R.B. 1995. An application of matrix modelling to growth prediction in Solomon Islands rain forest. A matrix projection model for Solomon Islands tropical rain forest was developed from a very restricted set of plot data collected on Kolombangara. Revisions were made to estimate ingrowth and mortality. The model was modified to provide predictions which approach climax forest in the Allardyce region. The model was used to develop predictions of growth rates in Allardyce forests. Limitations of the model and their effect on predictions are discussed. The predictions indicated that currently non-harvestable forest could be suitable for harvesting in 30 years, and that forest harvested with a high degree of care could be available for further harvesting in 40 years, provided that management practices are diligently conducted.

Key words: Tropical - growth - matrix projection - Solomon Islands

TENNENT, R.B. 1995. Aplikasi model matriks kepada taksiran tumbesaran hutan hujan di Kepulauan Solomon. Model unjuran matriks untuk hutan hujan tropika Kepulauan Solomon telah diasaskan berdasarkan set data plot yang terhad yang diambil di Kolombangara. Pemeriksaan semula telah dibuat untuk menganggar tumbesaran dalam dan kematian. Model tersebut telah diubahsuai untuk memberi taksiran kepada hutan klimaks di kawasan Allardyce. Model ini digunakan untuk membuat taksiran kadar tumbesaran di hutan-hutan Allardyce. Kelemahan model ini dan kesannya ke atas taksiran dibincangkan. Taksiran menunjukkan bahawa hutan yang belum boleh ditebang adalah sesuai untuk ditebang dalam masa 30 tahun. Hutan yang ditebang dengan cermat boleh ditebang semula dalam masa 40 tahun dengan syarat pengurusannya dijalankan dengan baik.

Introduction

In 1991, the Forestry Division of the Solomon Islands, Ministry of Natural Resources began work on a National Forest Resources Inventory. During the course of the inventory, the need arose for a set of growth projections 30 years hence for the forests of the Allardyce region of the Solomon Islands. The projections were required for two forest types: currently non-harvestable forest, and selectively harvested forest.

There was insufficient time to collect a suitable data set since the projections were needed within a one-month period. The existing data had a considerable number of limitations, which meant that methods had to be developed to circumvent the data limitations.

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This paper describes the procedures followed in developing the growth projections, and the limitations encountered during these procedures. The paper highlights the extent to which a limited data set can be used to develop a model.

Background

The Solomon Islands National Resources Inventory was carried out by the Forestry Division of the Ministry of Natural Resources, with the assistance of the Australian Government. The background to the inventory is given by King (1991). The inventory is described by Hammermaster (1990) and Tennent (1991), and the results are reported in Fearnside (1991), and Solomon Islands Forest Resources Inventory Project (SOLFRIP) (1992).

The inventory included two-stage forest sampling, and the development of volume tables (Tennent 1992). During this process, the absence of a suitable method for projection of forest growth was identified (Shield 1992). A review of the inventory called for an examination of growth in the Allardyce region, and the estimation of yield thirty years hence.

The most suitable existing data set has been collected in Kolombangara (Whitmore 1989). Here, experimental sample plots had been measured regularly since the 1960s, with the most recent measurements taken in 1991.

Solomon Islands rain forest

In the early 1970s approximately 84% of the Solomon Islands was covered in forest, with 4.5% lowland rain forest, and 63.3% hill rain forest (Hansell & Wall 1974). The lowland and hill rain forests have complex structures and composition, including a wide range of species, with elements of freshwater swamp forest characterised by *Terminalia brassii.* Montane forest occurs on higher altitude ridge tops and mountain summits, usually over 1000 m (SOLFRIP 1992).

The forest includes a range of canopy classes, often reflecting past cyclones and anthropogenic activities such as clearing, cultivation, and timber extraction. SOLFRIP (1992) includes the following description :

"A typical appearance on air photographs is a mid-dense forest with a canopy pattern that has scattered large crowns overtopping a gap-filling, even-aged canopy of smaller diameter crowns. Such areas are marginal between canopy density classes 2 and 3, although they are classified as having a density class of 3 if the remnant primary canopy coverage is more that 40 - 50%. Such areas are further classified in stratification level three as having a canopy comprising various crown sizes .... indicating the disturbed nature of the forest."[1]

[1]Canopy density class one indicates forest degraded by wind, slip or clearing. Canopy density class two indicates severe to moderate disturbance by cyclone, slip or logging. Canopy density class three indicates moderate disturbance.

Growth projections

The tropical rain forest, which is now recognised as a dynamic rather than static forest, presents considerable challenges to the growth modeller. Due to its complex structure, difficulties in collecting data and long prediction periods, comparatively little research has been carried out on growth projection methods. However, the Solomon Islands National Resources Inventory required growth projections to be made. The alternatives were to resort to subjective estimations, or to attempt to produce growth projections by using the limited data available.

Vanclay (1989) recommends that predictions be derived from cohort modelling. However, the data available at the time the projections were required did not allow for cohort modelling. In the absence of suitable data, Usher's (1966) classic matrix projection model was selected as a basis for developing the predictions. Extensions to Usher's basic model have been made by several authors (e.g. Michie & Buongiorno 1984, Michie & McCandless 1986, Korsgaard 1989) so that ingrowth and mortality can be included. Harrison and Michie (1985) examined methods of projecting growth over periods differing from sample plot remeasurement intervals.

Model development

A matrix projection model is composed of a transition probability matrix which translates the state of a forest at a particular point in time into the state at a future point in time. In the case in question, the state of the forest was represented by a diameter distribution. Ideally, each major species should be represented by its own transition probability matrix, with the overall forest growth projection derived from the projection of each species. In such a model, the matrix coefficients would be dependent on species.

The transition probability matrix is designed to reflect the possible changes in the state of a tree. It is assumed that the tree will do one of the following:

- die[2]
- remain in the same diameter class
- grow into the next diameter class

There are a number of caveats which must be examined. The possibility of a tree growing into a higher diameter class than the next one is avoided by defining class widths and increment period in such a way that this is precluded. However, seedlings and sapling trees may be expected to grow into the lowest diameter class. This source of additional trees is termed ingrowth. Also, some tree species may undergo prolonged periods of quiescence, showing little or no growth.

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[2]Usher's model assumed that mortality occurred between periods. In this model mortality is assumed to occur within a period.

Both ingrowth and quiescence present a problem to the modeller. In an ideal data set, such trees would be identified, and treated separately. If they are not identified, estimates of growth and mortality will be biased.

Preliminary model

The available data were used to develop a basic matrix projection model, combining all species. The measurements had been collected over a period of 26 years, and provided a reasonable indication of the rate at which surviving trees had been growing. They were inadequate for the estimation of mortality, particularly in the smaller diameter classes, which contained a higher number of trees.

There were no data suitable for estimating ingrowth and the degree of quiescence experienced during times of suppression. These limitations were the major restrictions affecting model development. There was no time available for additional data to be collected.

As a preliminary step, a model was developed to project 10 cm class diameter distributions over five-year periods.

The model's structure is:

$$d_{t+1} = Pd_t$$

where

$$d = \begin{vmatrix} d_1 \\ d_2 \\ \\ \\ d_n \end{vmatrix}, P = \begin{vmatrix} a_1 + k_1 & k_2 & k_3 & k_4 & \dots & k_{n-1} & k_n \\ b_1 & a_2 & . & . & \dots & . & . \\ . & b_2 & a_3 & . & \dots & . & . \\ . & . & b_3 & a_4 & \dots & . & . \\ \vdots & \vdots & \vdots & \vdots & . & \vdots & \vdots \\ . & . & . & . & . & a_{n-1} & . \\ . & . & . & . & . & b_{n-1} & a_n \end{vmatrix}$$

and t represents time.

The matrix d is a vector containing the number of trees, d_i, in diameter class i and the transition probability matrix, P, is composed of elements a_i, b_i, and k_j where a_i represents the probability of a tree remaining in diameter class i, b_i represents the probability of a tree making the transition to diameter class i, and k_i is a coefficient representing the influence on ingrowth of diameter class i.

Elements a_i, b_i, and d_i are defined as non-negative, with $a_i + b_i \le 1$. Elements k_i may take any value.

The values of element b_i were estimated from the average growth rate[3] of each class. For example, if the average growth of surviving trees in class *j* was 5 cm in a 5-year period, the value of b_j would be 0.5.

Given b_j, the value of a_j was calculated after estimating the average mortality for the diameter class. For example, if 5% of trees in class *j* had died by the end of the period, $a_j+b_j = 0.95$, and hence a_j would be .45. The percentage mortality was interpolated for those classes with insufficient data to allow an accurate estimate to be calculated.

This is simplistic, because the question of quiescence is not considered. The model can be expected to be biased, since the degree of quiescence can be expected to vary with forest structure. Forests with a high number of large-canopy trees will have a higher degree of quiescence than forests with a more open canopy.

Buongiorno and Michie (1980) developed a formula for estimating k_i. Their expected ingrowth function has the following form:

$$I_t = \beta_0 + \beta_1 \sum_{i=1}^{n} B_i (y_{it} - h_{it}) + \beta_2 \sum_{t=1}^{n} (y_{it} - h_{it})$$

where

I_t	is the expected ingrowth in time *t*
B_i	is the basal area of a tree in class *i*
y_i	is the number of stems in class *i*
h_i	is the number of stems harvested from class *i*

In this formulation β_0, β_1, and β_2 are constants expected to be positive, negative, and positive respectively.

There were insufficient data for their methodology to be applied without some modification. The h_i terms were set to zero, thus basing the estimation of ingrowth on the population of the diameter class at the start of the prediction period. A multiple regression was calculated to estimate the values of β_0, β_1, and β_2 from the limited data available. The coefficients produced did not meet the theoretical assumptions, with β_1 estimated to be positive, and β_2 negative.

As the multiple regression coefficients were theoretically unsound, an iterative approach was used to estimate suitable coefficients. The model was run using the estimated β_2 regression coefficients. The initial values of β_0, β_1, and β_2 were adjusted until the ingrowth predicted approximated the ingrowth observed. The absence of data prevented a more rigorous estimation of the ingrowth coefficients, and a full examination of the Buongiorno and Michie method.

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[3]Recent work (McDill & Amateis 1993) has examined more suitable methods of estimating parameters by using simple averages.

Model validation

The model was run and compared with the available data. As expected, the lack of mortality data and data defining expected periods of little or no growth produced stand structure predictions with excessive numbers of large-diameter trees. As a result, the ingrowth estimates were extremely unstable, with ingrowth tending to reflect positive feedback from the over-prediction of growth and the under-prediction of mortality.

The poor estimation of ingrowth was not a primary concern for the intended use of the model. It can be assumed that for short-term predictions (30 years in this investigation), any trees suitable for harvest at the end of the period would be well established at the beginning (Valentine & Furnival 1989). This reduced the main difficulties with mortality and quiescence.

The mortality coefficients were initially replaced with coefficients derived by fitting a smoothing function through the existing data. All coefficients were replaced with their predicted values. The model was then run, and the predicted mortality compared with the available observed mortality. The result was an over-prediction of mortality. All coefficients were adjusted by moving the smoothing function down, to provide a set of coefficients which produced predicted mortality closer to the observed mortality.

Model dampening

The Solomon Islands National Resources Inventory required growth predictions for the availability of timber in the future. For such a use, conservative estimates are required. Data limitations produced a tendency for the model to over-predict, which could have had dire consequences, and a dampening effect was therefore introduced to limit the model's predictions to biologically realistic levels.

Diameter distribution data for forest areas showing the highest standing volume in the Allardyce inventory were used to develop a hypothetically ideal climax forest diameter distribution. This distribution curve was smoothened, and set as an upper limit.

The model was modified to permit the number of stems in any diameter class to approach the hypothetical maximum for the class. If the number of stems in a diameter class was predicted to increase above the limit, the number of stems growing into that diameter class was constrained to the hypothetical limit. Any diameter classes containing more stems than the hypothetical maximum at the outset were only permitted to decrease or to remain static in terms of stem number. Because such diameter classes were invariably below 40 cm, the model's tendency to over-predict growth ensured that these classes decreased in number without the restriction applying.

Ingrowth was constrained to restrict the population of the lowest diameter class to the hypothetical maximum number.

The final form of the model provided estimates that were broadly representative of data from tropical forest that had been disturbed by a cyclonic event about 20 years previously[4].

Figure 1 shows the model's predictions for a test dataset.

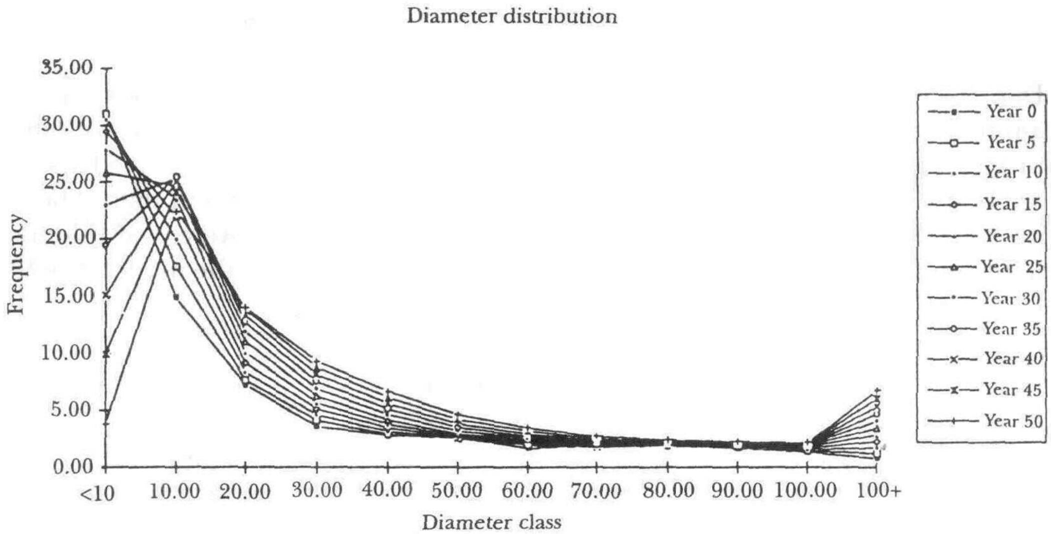

Figure 1. Model predictions for test dataset

Table 1 shows the model predictions used in Figure 1.

Table 1. Model diameter class frequency predictions for test dataset

Year	Diameter class (cm)											
	<10	10	20	30	40	50	60	70	80	90	100	100+
0	31	15	7	4	3	3	2	2	2	2	1	1
5	25	19	8	4	3	3	2	2	2	2	2	1
10	21	21	8	5	3	3	2	2	2	2	2	2
15	17	22	9	5	4	3	2	2	2	2	2	2
20	14	22	10	6	4	3	2	2	2	2	2	3
25	12	22	11	6	4	4	3	2	2	2	2	3
30	10	21	12	7	5	4	3	3	2	2	2	4
35	9	20	13	7	5	4	3	3	2	2	2	5
40	8	19	14	8	6	5	3	3	2	2	2	5
45	7	18	15	8	6	5	4	3	2	2	2	6
50	7	17	11	8	7	6	4	3	3	2	2	7

[4]This applies to Allardyce, Cyclone Ida in 1972, and Kolombangara, Cyclone Annie in 1969.

The predictions indicated a change from a forest with a few large trees and a much greater number of small trees to one dominated by large trees. Individual diameter classes show an initial increase in population, followed by a later decrease. These changes are representative of the change in structure that may be expected in an actual forest under similar circumstances.

Model error limits

Due to the data limitations and subsequent development of the model using approximation techniques, statistical error terms cannot be placed in the model prediction. Error estimates of simulation models such as the one described above are best derived by comparing model predictions with independent data. There were insufficient data for model development, and thus none were available for model validation. The projections obtained through the use of the model should be considered indicative only.

Yield prediction

Two predictions were prepared using the model. These were an estimate of the time required after logging for currently-harvestable forest to recover to the initial volume, and an estimate of the time required for forest with a history of harvest or cyclone damage (i.e. currently non-harvestable) to reach a harvestable condition. Volume was derived from tables developed by Tennent (1992) which predict volume from diameter, for all species.

Inventory data from the Allardyce region showed that the harvestable forest had an average standing volume of 81.8 m^3 on an average of 15.8 stems ha^{-1} of 60 cm-and-above diameter. This element of the diameter distribution was removed to simulate harvesting, and 20% of all lower diameter classes (a further 21.8 trees ha^{-1}) was removed to simulate logging damage. This represented a high standard of logging; actual logging damage often exceeds 20%.

The data showed that currently non-harvestable forest had an average standing volume of 37.8 m^3 on an average of 7.4 stems per ha^{-1} of 60 cm-and-above diameter.

The model was used to predict the growth of both types of forest.

Table 2 shows the yield predictions calculated for currently harvestable forest, and Table 3 for currently non-harvestable forest. The values shown are derived from only those stems over 60 cm dbh.

The predictions were broadly representative of the forest under consideration. Tables 2 and 3 indicate that the harvested forest will grow more rapidly than the currently non-harvestable forest. This would be expected. However, further examination of Table 2 indicates that growth rate would still be increasing after 50 years. This is unlikely, and indicates model instability.

Table 2. Yield predictions for currently-harvestable forest after logging

Year	Stems ha^{-1}	Volume (m^3)
0	0.0	0.0
5	2.1	6.5
10	4.3	14.1
15	6.6	22.9
20	9.3	33.5
25	11.2	43.1
30	13.1	53.4
35	14.9	64.3
40	16.7	75.9
45	18.3	87.4
50	19.4	95.7

Table 3. Growth prediction for currently non-harvestable forest

Year	Stems ha^{-1}	Volume (m^3)
0	7.4	37.8
5	8.8	44.5
10	11.2	54.5
15	13.5	65.1
20	14.9	73.2
25	16.4	81.5
30	17.5	87.5

Sensitivity analysis

The predictions derived are based on the model as developed, and as such cannot be given error limits. The model was run with slight changes to the assumptions used to derive the predictions, in order to test the sensitivity of the model to such changes.

For the currently harvestable forest, the logging damage was assumed to remove 40% of stems below 60 cm. The model again predicted 19.4 stems 60 cm and above at age 50, with a volume of 95.7 m^3. This indicates that the model is insensitive to the number of stems in the lower diameter classes.

For the currently non-harvestable forest, the model was run with the dampening mechanism removed. The unrestrained model predicted 19.8 stems of 60 cm and above at age 30, with a volume of 98.6 m^3. This indicates that the model has a tendency to over predict, if unrestrained.

These two examples show that the model required further work, which would require the collection of additional data.

Prediction restrictions

The predictions indicated that the currently-harvestable forest may be reharvested after approximately 40 years, and that the currently non-harvestable forest could be harvested in approximately 30 years, based on a minimum harvesting volume of 75 m^3 on 17 stems ha^{-1}.

Although the tendency of the model to over-predict has been reduced, theabove predictions are indicative only. It is unlikely that growth will exceed the predictions. If logging opens the canopy excessively or greatly reduces the number of viable medium-sized trees, pioneer species and non-timber species will take over the site and severely reduce the growth rate of timber trees. The predictions assume that logging will be carried out with the highest degree of caution, and that damage to non-crop trees is minimal. If logging damage is not controlled, the predicted volumes will not be realised.

The predictions represent an average site and an average species mix. The model was constructed using data from Kolombangara, and calibrated against general inventory data. Kolombangara soils are derived from volcanic parent material, while Allardyce soils are derived from non-calcareous sediments. The predictions should be considered in this light. If further data were available, a model could be prepared to relate predictions to specific sites and mixtures of species.

A further question to be considered is the likelihood of changes in species composition after any disturbance. No data were available and predictions from the model would have to be considered with this in mind. Such predictions would be more suitable for forests where post-harvest management aimed at producing loggable forest, and not intermediate forest types.

Predicted values should be considered to represent the maximum possible growth and productivity that could be achieved under diligent management, and careful logging. If management was not conducted to the highest degree, and if logging was not carried out with care and attention, the predicted yields obtained from the model are unlikely to be achieved.

Conclusion

Matrix projection methods are relatively simple to understand, and can be applied with the help of modern spreadsheet technology. In the example described, a model was developed from a very limited dataset and predictions were derived within a very short space of time.

The example given has considerable weaknesses, but demonstrates how the methodology could be adapted to accommodate the data limitations. The model was far from robust but the estimates derived had a degree of objectivity. Provisional rotation lengths, and cutting cycles within these rotations, could be proposed based on indications provided by the model. Further research is needed to support the model's predictions.

The investigation highlighted the need for additional growth data, especially for estimation of mortality, ingrowth, and quiescence. Future model development should include mortality data so that a more robust and objective method for predicting mortality could be derived. Trials aimed at collecting data on the extent to which ingrowth is generated by partial felling should be considered. An investigation into the time period during which individual species can remain quiescent is needed. Trials should ensure that suitable data are collected or located, in order that these aspects can be examined quantitatively.

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