# AN EVALUATION OF SOME WHOLE STAND-LEVEL MODELS FOR PREDICTING MULTI-SPECIES BASAL AREA IN A TROPICAL RAIN FOREST

**J.S.A. Osho**

*Department of Forest Resources Management, University of Ibadan, Ibadan, Nigeria*

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**OSHO, J.S.A. 1997. An evaluation of some whole stand-level models for predicting multi-species basal area in a tropical rain forest.** Three models, Weibull distribution, stage-structured matrix and multiple regression models, were used to predict the basal area of a tropical rain forest. The results from the models showed that basal area prediction from variables that emerge from whole stand-level of organisation was feasible. The Weibull distribution model predicted 34.82 $m^2$ $ha^{-1}$ for the forest stand, the matrix model 36.73 $m^2$ $ha^{-1}$ while the multiple regression model predicted a mean basal area of 26.70 $m^2$ $ha^{-1}$. There were varying levels of accuracy of basal area estimates among the models. The matrix model produced better fit in the larger diameter classes while the Weibull distribution model consistently under-estimated the basal area. It was noted that significant correlation coefficients between Weibull parameters and stand factors like stem density and mean diameters were partly responsible for the errors in the estimates. An adjustment value for the bias in basal area estimates by the Weibull model was obtained.

Key words: Weibull distribution - matrix model - basal area

**OSHO, J.S.A. 1997. Penilaian ke atas beberapa model aras dirian keseluruhan untuk meramal luas pangkal berbilang spesies di hutan hujan tropika.** Tiga model termasuk model-model taburan Weibull, matriks struktur peringkat dan regresi berganda digunakan untuk meramal luas pangkal hutan hujan tropika. Keputusan daripada model-model di atas menunjukkan bahawa ramalan luas pangkal daripada pemboleh ubah yang terbit daripada aras dirian keseluruhan organisasi dapat diterima. Model taburan Weibull meramalkan 34.82 $m^2$ $ha^{-1}$ untuk dirian hutan, model matriks 36.73 $m^2$ $ha^{-1}$ manakala model regresi berganda meramalkan purata luas kawasan 26.70 $m^2$ $ha^{-1}$. Terdapat ketepatan yang berbeza-beza untuk taksiran luas pangkal menggunakan model tersebut. Model matriks menghasilkan takikan yang lebih baik dalam kelas diameter yang lebih besar. Bagaimanapun model taburan Weibull sentiasa membuat taksiran yang rendah untuk luas pangkal. Koefisien korelasi yang ketara antara parameter Weibull dan faktor dirian seperti ketumpatan batang dan purata diameter sebahagiannya menyebabkan kesilapan dalam taksiran. Nilai penyelarasan bagi pincang dalam anggaran luas pangkal oleh model Weibull diperoleh.

## Introduction

Whole stand-level models which represent forest dynamics by mathematical equations with variables that emerge at the forest stand-level of organisation are increasingly being criticised for growth modelling in tropical forests. Some of these variables include tree density, minimum and maximum stand diameter, growth rate, regeneration/recruitment and tree mortality. Most modellers contend that such models have limited utility as it is hard to describe the forest adequately with such few variables. The chaotic theory (May 1976) with its arguable deductions seemed to have ruled out the use of such models for long-term predictions in ecology. More attention is now being focussed on individual-tree based models. The individual-tree based models attempt to simulate the dynamics of a forest by computing changes in each individual tree in a forest stand.

Most individual-tree based models tend not to be mathematicaly rigorous but fuzzy, leading to the accumulation of predigious quantities of data. The statistical analysis and interpretations of such data will no doubt present enormous problems. The complexities in tropical forests, arising largely from multi-species of indeterminate ages coupled with a wide range of life-forms make it all but impossible to collect complete data on all individual tress in a forest stand. While individual-tree based models tend to be more concerned with explanations of succession from theoretical approach, whole stand-level models concentrate on the modelling of forest dynamics for applied prediction and management purposes. In spite of the euphoria generated by individual-tree based models, there is little evidence of their adoptions particularly in natural tropical rain forests.

In natural tropical rain forests, where selective logging is a major forest management technique, the importance of recreating a specified diameter distribution or stand structure at the end of each cutting cycle is required for further existence. Weibull distribution models have been used to quantify the diameter distributions in even-aged forests (Bailey & Dell 1973, Smith 1980, Okojie 1981). Results of these predictions by Weibull function showed varying degrees of accurate characterisation of diameters.

Unfortunately, the results of these studies are not easily extended to uneven-aged natural tropical rain forests where the conditions of strict randomness are not met. Recently, more attention is being paid to the applications of Weibull functions to model growth in uneven-aged forests.

The main aim of this study was the application of Weibull distribution model to predict multi-species basal area for a natural tropical rain forest in southwestern Nigeria. I will critically appraise this modelling approach and compare its predictive potentials with tree population projection matrix model and regression model as tools for forest management. The length of cutting cycle in the Nigerian rain forests is about 50 years. The use of whole stand-level models for predicting tree population dynamics within this short range can safely be investigated for forest management purposes.

## Methods

### *The study site*

The forest data sets used in this study came from a permanent sample plot (PSP.84) of 1.47 ha located in Idanre Forest Reserve (6°50'N, 5°10'E). This is a lowland rain forest zone in southwestern Nigeria. The vegetation is a mixed moist evergreen secondary high forest with complete canopy and well developed storeys. Detailed information on vegetation and soil-type is available in the works of Redhead (1971) and Hall (1977). The plot was further subdivided into 16 square subplots of 33 × 33 m and complete enumerations and measurements of mapped trees took place in 1956, 1963, 1969 and 1974. The data sets provided full and complete information on live trees and their diameter increments, recruitment and mortality. About 100 tree species were identified in the plot (Osho 1991).

All the trees from each inventory were pooled and classified into six diameter size classes $D_i$, $i = 1 \ldots 6$ such that $D_1 < D_2 < D_3 \ldots < D_6$, each with a width of 10 cm.

$D_1 = (0 - 10\text{ cm})$, $D_2 = (10^+ - 20\text{ cm})$, $D_3 = (20^+ - 30\text{ cm})$, $D_4 = (30^+ - 40\text{ cm})$, $D_5 = (40^+ - 50\text{ cm})$ and $D_6 = 50^+$cm over. The basal area distributions of the plot are shown in Table 1.

**Table 1.** Stand basal area ($m^2\ ha^{-1}$) distributions (1956-1974)

| | | Year | | | |
|---|---|---|---|---|---|
| | | 1956 | 1963 | 1969 | 1974 |
| | $D_1$ | 3.78 | 3.38 | 2.86 | 3.66 |
| | $D_2$ | 7.45 | 8.63 | 7.84 | 8.10 |
| Dbh(cm) | $D_3$ | 7.71 | 9.77 | 8.45 | 7.76 |
| | $D_4$ | 5.60 | 6.53 | 5.97 | 7.10 |
| | $D_5$ | 3.23 | 2.91 | 3.07 | 4.20 |
| | $D_6$ | 4.01 | 3.70 | 7.10 | 8.02 |
| Total | | 31.78 | 34.92 | 35.29 | 38.84 |

### *3 - parameter Weibull distribution model*

The form of 3-parameter Weibull distribution commonly used to quantify diameter distributions is given by

$$N_i = Nw(c/b)\ [(d_i - a)/b]^{c-1} \exp[-[(d_i - a)/b]^c]$$
$$d_i \geq a \geq 0;\ b, c, \geq 0$$

where

$N_i$ = number of trees in diameter size class $i$
$d_i$ = mean tree diameter in class $i$
$w$ = width of diameter class
$N$ = total number of trees in the stand

The parameters *a*, *b* and *c* represent the locations (minimum stand diameter), scale (a value for cumulative distribution) and shape of the distribution respectively.

*Estimation of the Weibull parameters (a, b and c)*

The method of maximum likehood would be used in the estimation of the Weibull parameters *b* and *c*. We would assume that the parameter *a* is known. The assumption on *a* does not in anyway lead to a loss of generality since in most forestry practical problems, *a* will always be known. For example, in selective logging, in the Nigerian rain forests, minimum diameter size is always prescribed.

Johnson and Katz (1970) gave the maximum likehood estimate of *b* and *c* as:

$$c = [(\Sigma x_i^c)^{-1} \log x_i) \ (\Sigma x_i^c)^{-1} n^{-1} \Sigma \log x_i ]^{-1} \quad (2)$$

$$b = (n^{-1} \Sigma x_i c)^{1/c} \quad (3)$$

where

$x_i$ = diameter of tree $i$

$n$ = sample size

When the value of $a$ is unknown its maximum likelihood estimator is given by $a$ = minimum ($x_i$, $x_2$ ......... $x_n$). Another equation given by

$$(c - 1) \ (x_i - a) = ca^c (x_i - a)^{c-1} \quad (4)$$

is added. Since the sets of equations 2, 3 and 4 cannot be obtained in the closed form, iterative solutions procedure based on Newton-Rapson's method was used (Osho 1989).

*Stage - structured growth matrix model (P)*

A stage-structured growth matrix model P(6 × 6) was developed and used to project the tree populations of the forest based on 18-year growth period (Osho 1991). The rows and columns of the model P represented the diameter size classes of the tree population. The first row of the matrix was given by

$$a_{ij} = p_{ij} + r_j, \ j = 1,2,...6$$

and the coefficients for all other rows were given by

$$a_{ij} = p_{ij} = (1 - m_j) s_j = \text{diagonal elements}$$

$$a_i + 1_j = p_i + 1_j = m_j s_j = \text{subdiagonal elements}$$

All other coefficients were zero.

$p_{ij}$ = contribution of trees in class $j$ to class $i$
$r_j$ = average recruitment per tree in class $j$
$m_j$ = probability of trees moving from class $j$ to class $j+i$ at the end of the growth period
$(1 - m_j)$ = probability of trees remaining in class $j$ at the end of the growth period

Stand evolution was projected over a 18-year transition period from the matrix equation.

$$n(t+1) = Pn(t)$$

where

$n(t)$ = initial diameter distribution in 1956
$n(t+1)$ = projected diameter distribution at the end of the growth period in 1974

The stand basal area was then obtained from the diameter characterisations given by the matrix equations.

*Multiple regression model*

A multiple regression model of the form

$$y_{ij} = \beta_0 + \beta_1 x_{1i}^{-1} + \beta_2 x_{1i}^{-1} x_{3i} + \beta_3 (1 - x_{1i}^{-1} x_{2i}) + e_{ij}$$

$y_{ij}$ = subplot basal area, $i = 1,2, \ldots 16$
$x_{1i}$ = number of live trees in subplot $i$
$x_{2i}$ = number of dead trees during the 18-year growth period
$x_{3i}$ = total recruitment into the tree population during the growth period

was fitted to the stand data. The plot was subdivided into 16 square subplots of 33 × 33 m and 100% enumeration of all trees with a minimum of 4.8 cm in diameter at breast height (dbh) was carried out for each subplot.

$\beta_0$, $\beta_1$ and $\beta_3$ represent regression coefficients to be estimated, and $e_{ij} \sim N(0, \sigma^2)$ the usual normality assumption of the error variable.

The estimates of the Weibull parameters *b* and *c* are shown below in Table 2.

**Table 2.** Estimates of Weibull parameters (*b* and *c*)

| Year | *b* | *c* | (Trees $ha^{-1}$) |
|---|---|---|---|
| 1956 | 14.3730 | 1.5137 | 1240 |
| 1963 | 15.0766 | 1.5078 | 1105 |
| 1969 | 15.5789 | 1.5366 | 1001 |
| 1974 | 14.6524 | 1.4682 | 1148 |

The growth matrix model P was given by

$$P = \begin{bmatrix} .42 & 0 & .73 & 2.24 & 5.04 & 7.42 \\ .06 & .62 & 0 & 0 & 0 & 0 \\ 0 & .05 & .66 & 0 & 0 & 0 \\ 0 & 0 & .14 & .79 & 0 & 0 \\ 0 & 0 & 0 & .20 & .62 & 0 \\ 0 & 0 & 0 & 0 & .1 & .89 \end{bmatrix}$$

The prediction equation for the regression model was given by

$$y_i = 166 + 587.2x_{1i}^{-1} - 9.0\, x_{1i}^{-1}x_{3i} + 8.5\,(1 - x_{1i}^{-1}\, x_{2i}) \quad (R^2 = 0.67)$$

## Results

### *Predictions with the models*

The predicted basal area of the forest stand from the Weibull distribution model and the growth matrix model is shown in Table 3. Also the predicted basal area of the forest at the subplot level by the regression model is shown in Table 4.

There were some agreements between the observed basal area and the predicted basal area of the forest stand. The Weibull distribution model underestimated the basal area from the largest diameter classes 5 and 6. The error rates in these merchantable size classes 5 and 6 were very high. The basal area estimates from the other lower diameter classes were more acurate. The error rates in the lower diameter classes were much lower than those from the large diameter classes. This error pattern would suggest that there might be some correlation between the Weibull parameters and stand diameter distribution. Also, variations in the values of the Weibull parameters followed the changes in the population density over time. When the stand's stem density declined from 1240 trees $ha^{-1}$ in 1956 to 1001 trees $ha^{-1}$ in 1969, the estimates of *b* and *c* increased. But when the tree population density increased to 1148 trees $ha^{-1}$ in 1974, the estimates of the parameters decreased correspondingly.

The matrix models predictions were very close to the actual stand basal area with greater precision for the larger trees from the merchantable size classes. Overall, the matrix model's predictions were closer to the actual values than the predictions from the Weibull distribution model. The superiority of the matrix model can probably be associated with the effect of diameter distribution itself on the growth of trees in the uneven-aged stand (Hansen & Nayland 1987). While the effects of the growth were accounted for in the matrix model, growth effects were not specifically treated in the Weibull distribution. Amongst the small-sized trees where growth rate was very low (0.002 cm $y^1$), the error rate in the predicted basal area for the larger trees with higher growth rate (0.2 cm $y^1$) remained at about 16.0% (see Figure 1).

**Table 3.** The predicted stand basal area by the Weibull and matrix models for 1974

| Dbh class | Observed ($m^2$ $ha^{-1}$) | Prediction (Weibull) | Error rate* (%) | Prediction (matrix) | Error rate* % |
|---|---|---|---|---|---|
| 1 | 3.66 | 3.55 | + 3.0 | 3.90 | - 6.6 |
| 2 | 8.10 | 9.35 | - 15.4 | 6.45 | + 20.4 |
| 3 | 7.76 | 8.45 | - 8.6 | 7.56 | + 2.6 |
| 4 | 7.10 | 6.72 | + 5.4 | 7.18 | - 1.1 |
| 5 | 4.20 | 3.07 | + 26.9 | 4.85 | - 15.5 |
| 6 | 8.02 | 3.70 | + 53.9 | 6.79 | + 15.3 |

$$\text{*Error rate} = \frac{\text{observed-predicted}}{\text{observed}} \times 100$$

**Table 4.** The predicted stand basal area by the multiple regression model

| Subplot | Observed | Predicted | Error rate |
|---|---|---|---|
| 1 | 2.02 | 1.64 | + 18.81 |
| 2 | 1.59 | 2.05 | - 28.93 |
| 3 | 2.48 | 2.46 | + 0.81 |
| 4 | 1.68 | 1.94 | - 15.48 |
| 5 | 2.02 | 2.36 | - 16.83 |
| 6 | 2.28 | 2.60 | - 14.04 |
| 7 | 2.60 | 2.72 | - 12.00 |
| 8 | 2.13 | 2.10 | + 1.40 |
| 9 | 2.74 | 2.87 | - 4.74 |
| 10 | 2.58 | 2.33 | + 9.69 |
| 11 | 3.49 | 2.85 | + 18.34 |
| 12 | 3.14 | 2.79 | + 11.15 |
| 13 | 2.09 | 1.94 | + 7.18 |
| 14 | 2.49 | 2.85 | - 14.46 |
| 15 | 2.77 | 2.68 | + 3.25 |
| 16 | 2.74 | 2.66 | + 2.92 |

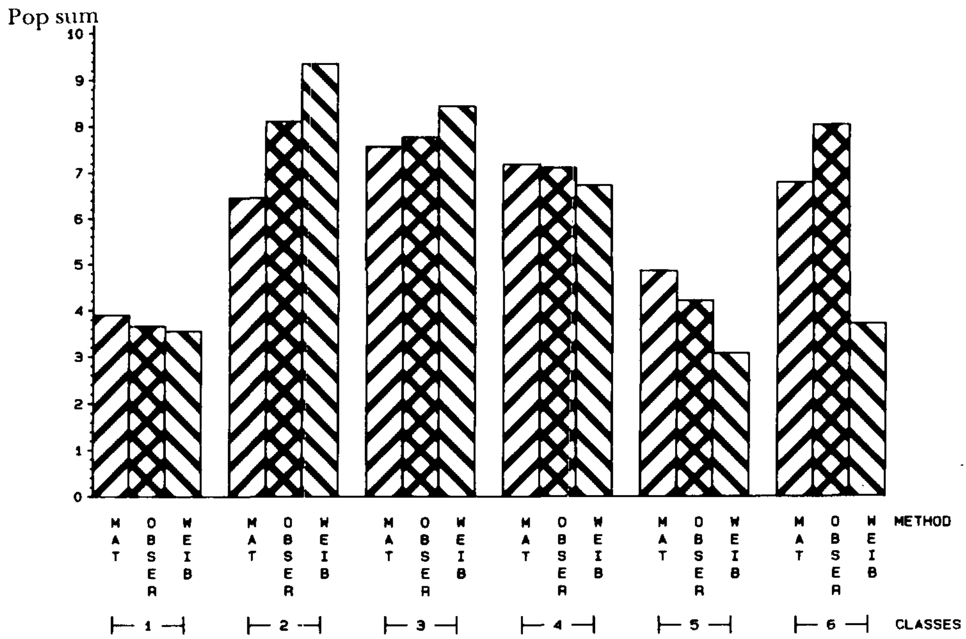


**Figure 1.** The plot of the observed and predicted basal areas of the forest stand by the Weibull and matrix models

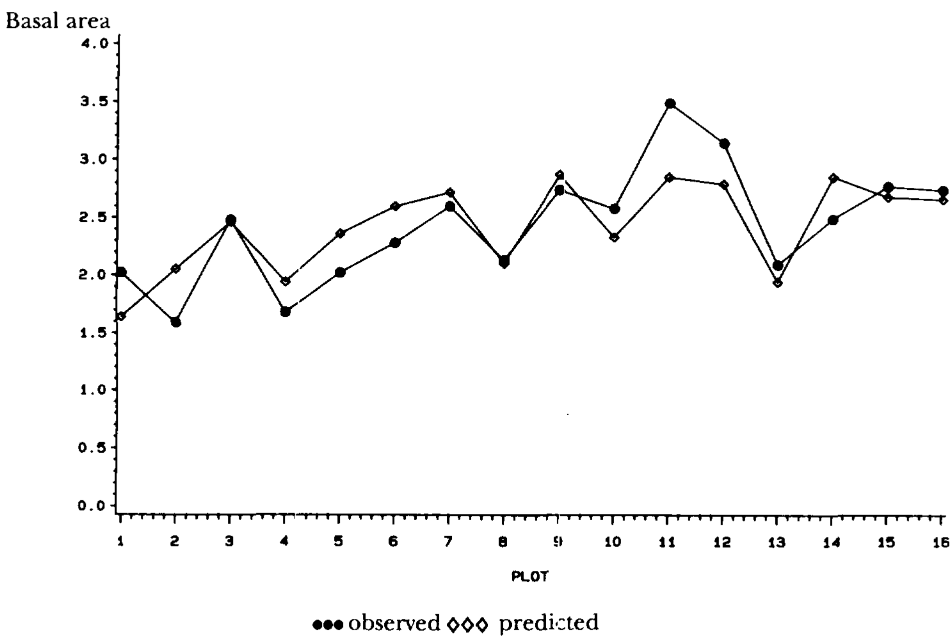


**Figure 2.** Graph of the observed and predicted stand basal areas by the multiple regression model

The prediction from the regression model was not characterised into diameter size classes as it was done in the other two models but only into subplots. The regression model accounted for about 67% ($R^2 = 0.67$) of the variations in the basal area of the stand. The predicted basal area was similar to the measured basal area (see Figure 2). Except for one or two subplots, the error rate in predicting basal area was lower when compared with the Weibull model and almost the same with the error rate in the matrix model.

### *Correlation between Weibull parameters and the stand variables*

Due to the observed movement of the Weibull parameters *b* and *c* with the stand's stem density and the pattern of the prediction errors, it was decided to investigate the relationship between these parameters and the stand's stem density, minimum and maximum diameters. Random samples were made from the stand data and were used to estimate correlation coefficients (see Table 5).

**Table 5a.** Weibull parameters estimates from random samples

| *b* | *c* | Stem density (trees ha$^{-1}$) | Minimum diameter (cm) | Maximum diameter (cm) |
|---|---|---|---|---|
| 16.6160 | 1.3528 | 59 | 6.43 | 64.11 |
| 16.4096 | 1.4177 | 117 | 6.64 | 61.91 |
| 15.0561 | 1.3977 | 201 | 6.55 | 61.00 |
| 14.8954 | 1.4287 | 252 | 6.53 | 61.00 |
| 14.8159 | 1.4571 | 324 | 6.68 | 59.44 |
| 14.2005 | 1.4777 | 421 | 6.61 | 63.45 |
| 14.4294 | 1.4554 | 465 | 6.51 | 63.45 |
| 14.3577 | 1.4716 | 531 | 6.59 | 63.45 |
| 14.0524 | 1.4777 | 631 | 6.54 | 62.40 |
| 14.0061 | 1.4906 | 705 | 6.60 | 62.40 |
| 13.9304 | 1.4906 | 785 | 6.20 | 61.27 |
| 14.0340 | 1.5001 | 855 | 6.64 | 63.27 |
| 14.0778 | 1.5067 | 936 | 6.63 | 61.23 |
| 14.1028 | 1.4831 | 1023 | 6.64 | 63.27 |
| 14.2212 | 1.4881 | 1098 | 6.68 | 62.67 |
| 14.4319 | 1.4819 | 1186 | 6.73 | 61.96 |
| 14.3694 | 1.4865 | 1275 | 6.74 | 61.96 |
| 14.3878 | 1.4857 | 1360 | 6.72 | 61.16 |
| 14.4572 | 1.4970 | 1441 | 6.72 | 61.17 |
| 14.4262 | 1.5006 | 1492 | 6.74 | 61.17 |
| 14.3589 | 1.4994 | 1578 | 6.74 | 61.17 |
| 14.4301 | 1.5026 | 1667 | 6.74 | 61.17 |
| 14.3700 | 1.5137 | 1822 | 6.74 | 61.17 |

**Table 5b.** Correlation coefficient between Weibull parameters and stand factors

| Factor | $b$ | $c$ |
|---|---|---|
| Stem density | -0.52 | 0.78 |
| Minimum diameter | -0.14 | 0.42 |
| Maximum diameter | 0.21 | 0.28 |

The parameter $b$ was significantly and negatively correlated with stem density. The correlation of $b$ with minimum diameter and maximum diameter was poor but improved with mean diameter. On the other hand, the parameter $c$ was highly and positively correlated with the stem density. The relationship between $c$ and the minimum diameter improved over the value obtained for $b$ but remained rather poor.

The significant correlation between the Weibull parameters and the stand factors could be used to correct the bias of the estimate of the basal area in the two largest diameter classes 5 and 6.

A suggested function for estimating the bias was given as

$$\text{bias} = \frac{1}{p}(r_a^2 + r_b^2 + r_c^2)^{\frac{1}{2}} B_i$$

where

$r_a$ = correlation coefficient between Weibull parameter $a$ and stem density
$r_b$ = correlation coefficient between Weibull parameter $b$ and stem density
$r_c$ = correlation coefficient between Weibull parameter $c$ and stem density
$B_i$ = estimate of basal area in the diameter class $i$, $i$ = 5 and 6
$p$ = number of parameters in the fitted Weibull model

The estimate of the bias is strictly positive and should be used to adjust for the underestimation in the two largest diameter classes 5 and 6 upwards. However, the adjustment is recommended only for significant correlation values.

## Discussion and conclusion

The underestimation of the basal area in the merchantable diameter classes severely limits the use of Weibull distribution models in uneven-aged stands. Serious deficiencies could occur if forest management practices are developed, based on such characterisation for all size classes.

The matrix model performed much better than the Weibull model particularly in the merchantable classes. Since more emphasis is naturally placed on the larger diameter classes in the formulation of harvesting regimes from rain forests, matrix models are likely to contribute significantly to the development of harvesting schedules than the Weibull model.

The predictions from the regression model are considered biologically reasonable, given the nature of the explanatory variables. These variables can easily be measured in the forest, unlike other variables like crown size and height. In natural tropical forests, height measurement is often difficult, very expensive and invariably inaccurate. An immediate advantage of the technique will probably be a rapid development of the model which might enhance its adoption.

The importance of the determination of forest yield cannot be overemphasised. In Nigeria, a lot of pressure is being put on the forest resources manager by the government to provide information on the expected revenues from the rain forests. These models could be used for quick estimate of the timber yield from which the expected revenues could be derived. Forest management in Nigeria is extensive. We lack up-to-date inventory data on standing tree crops and exploitation. This is probably due to inadequate infrastructure and the complexities inherent in tropical rain forests. These models could provide some of the stand attributes rather simply and cheaply thus laying the foundation for the formulation and development of intensive forest management practices in Nigeria.

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