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A SIMPLE GRAPHICAL METHOD FOR COMPARISONS OF MORTALITY, RECRUITMENT AND OTHER COUNT DEFINED 'EVENT-RATES'

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Researchers frequently compare rates of change based on counts (e.g. mortality, germination, infection, recruitment). The estimation of mortality and recruitment rates from tree census data has been discussed elsewhere (Sheil 1995, Sheil *et al.* 1995, Sheil & May 1996). However, rather than estimating absolute rates, researchers may often need to determine whether two rates are significantly different. Most biometric texts and computer packages are not very helpful on this question (but see Manugistics Inc. 1993), and guidance is not readily sifted out of the more technical literature (e.g. Dobson 1990). This note is intended to address three related issues: 1) to inform researchers who are not familiar with the appropriate tests, 2) to present an approach that avoids analytical complexity, and 3) to provide some feel for the power of the test. This third point should assist in experimental design by providing guidance on the sample sizes and differences required for significance to be detected.

The comparison of integer rate counts can take many forms. A common example is in comparing demographic rates such as the gain and loss of members of a population (e.g. stems). For example, if counted gains (recruitment) are significantly different to counted losses (mortality), the observed population is unlikely to be at equilibrium. The equality of two such rates can be tested using 'comparison of two Poisson-counts' (Mansfield 1980, Dobson 1990). A full account of the reasoning and analysis behind the test requires relatively formidable mathematics, and will not be attempted here (see Birch 1963, Mansfield 1980, Dobson 1990). The intention here is to give some guidance on the use of the method and show how complex analysis procedures can be avoided.

The comparison of two Poisson-counts assumes that each count represents a single independent event. Mortality from tree fall damage and recruitment associated with canopy gaps indicate that this is not biologically realistic; however, the approach is useful in exploratory evaluations by providing guidance to possible levels of significance (minimum achievable *p* value). The method is appropriate both for comparisons of unbounded counts (e.g. recruitment) and also bounded (i.e. binomial) contrasts (e.g. mortality, seed germination, infection: Birch 1963, Dobson 1990). The method also allows for the comparison of mortality with recruitment—a contrast which might at first sight raise conceptual difficulties given that one is 'bounded' and one is 'unbounded'. Two further types of count-rate data can be distinguished: 1) census derived data (i.e. from evaluations at the start and end of a known period); and 2) directly observed events over a known period (the observations may thus include transient population members, see Sheil & May 1996). While both census and observation counts can be addressed directly through the comparison of two Poisson-counts method, they cannot be combined within a comparison.

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Using the Poisson-counts method for comparing the counts *X* and *Y* we assess the null hypothesis that the observed event-rates are equal [$X_{Rate} = Y_{Rate}$] and derive a probability value, *p*, that this is true. In the commonly encountered case of equal observation (or census) periods and equivalent samples (e.g. in a defined area, or a number of pre-marked individuals), the technique loses complexity and becomes a function of only the two counts. In such cases the test is symmetric, so *X* and *Y* are interchangeable. Every possible pair of counts has an associated *p* value. A reference probability can be chosen (e.g. $p = 0.05$), and the counts, *Y*, that provide the actual *p* value nearest to the reference can be identified for each value of *X*. When these points are plotted, a relatively simple pattern is found on either side of the central diagonal. Although these 'near-critical' points have integer coordinates, the underlying pattern of *X* against *Y* is curvilinear. A simple model was devised where, for each reference *p*, *Y* can be viewed as a *continuous* function of *X* (actually two functions: one each for $X>Y$ and $X<Y$).

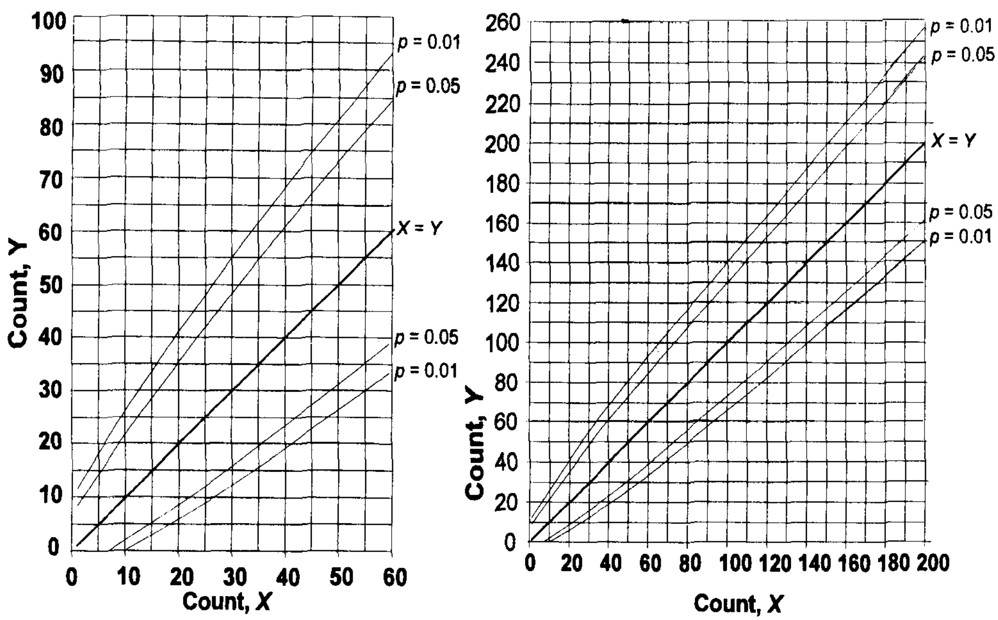

Figure 1. A graphical presentation of critical values ($p = 0.05$ and $p = 0.01$) for comparisons of two rate-counts. This graphical format can be used to judge whether two given counts (from an equivalent sample and period) are significantly different. Note the 'zone of non-significance' is the bounded area immediately to either side of the diagonal $X = Y$ line. The test assumes each event (count) occurs independently.

In Figure 1, the upper and lower critical values have been interpolated from calculated points for *p* values of 0.01 and 0.05. Quartic polynomials were found to provide a satisfactory fit, ($R^2>0.9999$, and all deviations were less than 0.5, total $n = 60$, with 10 test values). These continuous models are convenient, provide a visually accessible representation of the critical values, and most importantly can be generated without detailed analysis (e.g. in a spread-sheet). The approximate critical values, for *X* and *Y* up to 250, are as follows:

$p<0.05$,	if	$Y > -6\times10^{-8}X^4 + 3\times10^{-5}X^3 - 0.0049X^2 + 1.501X + 7.09$,
or		$Y < 2\times10^{-8}X^4 - 10^{-5}X^3 + 0.0027X^2 + 0.601X - 4.22$.
$p<0.01$,	if	$Y > -8\times10^{-8}X^4 + 4\times10^{-5}X^3 - 0.0067X^2 + 1.665X + 10.11$,
or		$Y < 2\times10^{-8}X^4 - 10^{-5}X^3 + 0.0031X^2 + 0.505X - 5.20$.

For counts up to 20 000 a more complex formulation is useful. Here if the identities of counts X and Y are allocated so that $Y>X$, then:

$p<0.05$,	if	$Y > 1.000022X + 2.76771X^{1/2} - 0.935(X-1)/(X+1) + 4.04$,
and		
$p<0.01$,	if	$Y > 0.999749X + 3.6957X^{1/2} - 0.459\mathrm{Log}_e X + 6.06$.

Again $R^2>0.9999$, ($n = 64$), and all residuals are less than 1 (including 10 test values). Unlike the polynomials both expressions show good general properties, e.g. errors apparently less than 0.001 and 0.02% respectively at X values 10 times beyond the fitted range ($X = 200\ 000$). Note that all p values provided here relate to the two-sided test.

I hope that the simple and accurate graphical models provided will be of utility to biologists who work with count-based changes. By summarising all possible counts and the regions of significant differences, these models should help in planning studies. The number of observations required to detect significant rate differences may surprise some researchers. An examination of the graphs will show that a significant result (at $p = 0.05$) between two counts differing by 100% requires that the contrasted counts only surpass 14 versus 28, a 30% difference must excede 100 versus 131, but a 10% difference requires almost 1000 counts for each (830 versus 913), while a 1% difference is only detectable near 80 000 counts (77 500 versus 78 275).

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