Journal of Tropical Forest Science 12(4):669–681 (2000)

MODELLING DIAMETER DISTRIBUTION IN EVEN-AGED AND UNEVEN-AGED FOREST STANDS

Kamziah Abd Kudus,

Faculty of Forestry, Universiti Putra Malaysia, 43400 Serdang, Selangor Darul Ehsan, Malaysia

M. I. Ahmad

Department of Mathematics, Faculty of Science and Environmental Studies, Universiti Putra Malaysia, 43400 Serdang, Selangor Darul Ehsan, Malaysia

&

Ahmad Zuhaidi Yahya

Forest Research Institute of Malaysia, 52109 Kepong, Selangor Darul Ehsan, Malaysia

Received June 1998

KAMZIAH, A. K., AHMAD, M. I. & AHMAD ZUHAIDI, Y. 2000. Modelling diameter distribution in even-aged and uneven-aged forest stands. Five distributions, Weibull, gamma, Johnson S_B, log normal and generalised normal, are compared in terms of their ability to model diameter data in even-aged and uneven-aged forest stands. Moments ratio diagrams of various statistical distributions are applied as a measure of the flexibility of the distribution in regard to their changes in shape. Some of the strengths and weaknesses of the distributions that have been used for describing diameter distributions in even-aged and uneven-aged stands are discussed. Data were obtained from 16 uneven-aged stands of mixed species located at Bukit Lagong Forest Reserve, Kepong, Selangor, and 9 even-aged stands of *Acacia mangium* species located at Kemasul Forest Reserve, Pahang. All the stands were from plantations and the ages ranged from 5 to 22 y.

Key words: Diameter distribution - maximum likelihood estimators - probability density function - loglikelihood function

KAMZIAH, A. K., AHMAD, M. I. & AHMAD ZUHAIDI, Y. 2000. Model taburan garis pusat bagi dirian hutan seumur dan tak seumur. Lima taburan, Weibull, gamma, Johnson S_B lognormal dan normal teritlak dibandingkan dari segi keupayaannya untuk membentuk model data garis pusat bagi dirian hutan seumur dan tak seumur. Gambarajah nisbah momen daripada pelbagai taburan statistik digunakan sebagai satu ukuran kelenturan berhubung dengan perubahan dalam bentuk sesebuah taburan itu. Beberapa kekuatan dan kelemahan bagi taburan-taburan yang pernah digunakan untuk menggambarkan taburan garis pusat bagi dirian seumur dan tak seumur ini dibincangkan. Data garis pusat diperoleh daripada 16 dirian seumur dan tak seumur daripada pelbagai spesies terletak di Hutan Simpan Bukit Lagong, Kepong, Selangor dan 9 dirian seumur daripada spesies *Acacia mangium* terletak di Hutan Simpan Kemasul, Pahang. Kesemua dirian tersebut merupakan hutan ladang dan umurnya dalam lingkungan 5 hingga 22 tahun.

Introduction

The distribution of diameter is the most simple factor to describe the properties of forest stands. Other variables such as volume, value, conversion cost, and product specifications are well correlated with diameter. Its relationship to site, stand composition, age and density is often valuable for economic and biological purposes. This quantitative information is helpful for managing forestlands in a sustainable manner.

For many years researchers have put considerable interest on describing the frequency distribution of diameter measurements in forest stands using probability density functions and have employed various distributions for both even-aged and mixed-aged stands with varying degrees of success (Zohrer 1972, Bailey & Dell 1973 and references therein, Clutter & Allison 1974).

According to Meyer and Stevenson (1943), in 1898 de Liocourt constructed a model based on the geometric progression for diameter distributions from uneven-aged forests and later on, Meyer and Stevenson applied this general model, the exponential distribution, to a forest of mixed species in Pennsylvania (Bailey & Dell 1973). Bailey and Dell (1973) note that other systems and distributions which broaden the consideration to include mound shape are Gram-Charlier series (Meyer 1930), the Pearl-Reed growth curve (Osborne & Schumacher 1935, Nelson 1964), the gamma distribution (Nelson 1964), and the three-parameter logarithmic-normal (Bliss & Reinker 1964). The beta distribution, which is essentially a reparameterisation of Pearson's more general Type 1, was applied to diameter distributions by Clutter and Bennett (1965) and later on McGee and Della-Bianca (1967), and Lenhart and Clutter (1971) subsequently developed the applications of models based on the beta distribution (Bailey & Dell 1973).

The main problem in fitting distributions has been the choice of statistical distribution function for describing the probabilities of interest (Hafley & Schreuder 1977). Hafley and Schreuder (1977) indicate that the criteria for choosing a distribution appear to be that the distribution be relatively simple to fit in terms of parameter estimation, sufficiently flexible to fit a relatively broad spectrum of shapes, lend itself easily to simple integration techniques for estimating proportions in various size classes, and fit any given set of observations well.

The probability density function should cover shapes of either positive or negative skewness. Bailey and Dell (1973) emphasised that any constant in the model should be easily related to shape and location features of the distribution and thus vary in a consistent manner with stand characteristics. The function should provide a promising base for advanced development and should also be easily fitted to observed data using parameter estimators that have desirable statistical properties.

An application of this method is the development of growth and yield systems. Diameter growth functions can be developed by applying the same form of probability density function (pdf) to the diameter distribution from the beginning until the end of a growing period (Kamziah 1998). Yield model can be derived based on approximation of a diameter distribution by a probability function. Numerous growth and yield systems based on the probability density function have since been developed (Smalley & Bailey 1974, Feduccia *et al.* 1979, Matney & Sullivan 1982, Baldwin & Feduccia 1987, Brooks *et al.* 1992). Hafley and Schreuder (1997) found that the Johnson S_B distribution is appropriate for both diameter and height, and was the driving force in a yield prediction model (Hafley *et al.* 1982).

Statistical distribution

Studies have been conducted to describe the frequency distribution of diameter measurements in forest stands using probability density function. In this study, we compare five distributions, Weibull, gamma, Johnson S_B, lognormal and generalised normal, in terms of their ability to model diameter data in even-aged and uneven-aged forests stands.

The probability density functions, likelihood functions and the loglikelihood functions for the five distributions are given as follows.

Weibull distribution

Probability distribution function:

$$f(x) = \frac{c(x-a)^{c-1}}{b^c} \quad \exp\left[-\left((x-a)/b\right)^c\right]$$

$$0 \le x < \infty \quad , \quad 0 \le a \le x \quad , \quad b, c > 0$$

where

a = location parameter
b = scale parameter
c = shape parameter

Likelihood function:

$$L = \frac{C^n \prod_{i=1}^{n} (x_i - a)^{c-1}}{b^{nc}} \quad \exp \quad \sum_{i=1}^{n} \left[-\left((x_i - a)/b\right)^c\right]$$

Loglikelihood function:

$$\mathrm{Log}\,(L) = n \log c - nc \log b + (c-1) \sum_{i=1}^{n} \log (x_i - a) - \frac{1}{b^c} \sum_{i=1}^{n} (x_i - a)^c$$

Gamma distribution

Probability density function:

$$f(x) = \left(\frac{x-a}{b}\right)^{c-1} \left[\exp\left(-\left(\frac{x-a}{b}\right)\right)\right] \Big/ b\Gamma(c)$$

$$0 \le x < \infty \quad , \quad 0 \le a \le x \quad , \quad b, c > 0$$

a = location parameter
b = scale parameter
c = shape parameter

where

$$\Gamma(c) \approx e^{-c}\, c^{(c-\frac{1}{2})} \quad (2\pi)^{\frac{1}{2}} \left[1 + \frac{1}{12c} + \frac{1}{288c^2} - \frac{139}{51840c^3} - \frac{571}{2488320c^4} + \ldots\right]$$

$$(c \to \infty \text{ in } |\arg c| < \pi)$$

Likelihood function:

$$L = \prod_{i=1}^{n} \left(\frac{x_i - a}{b}\right)^{c-1} \left[\exp \sum_{i=1}^{n} \left(-\left(\frac{x_i - a}{b}\right)\right)\right] \Big/ (b\Gamma(c))^n$$

Loglikelihood function:

$$\text{Log}\,(L) = (c-1)\sum_{i=1}^{n} \log\left(\frac{x_i - a}{b}\right) - \sum_{i=1}^{n}\left(\frac{x_i - a}{b}\right)$$

$$n \log b - n \log \Gamma\,(c).$$

Johnson S_B distribution

The Johnson S_B distribution has four parameters which are the lower limit, e, the upper limit, l and two shape parameters, g and d. For the purpose of this study, we assume e = 0 and therefore the distribution becomes a three-parameter distribution bounded by l from above.

Probability density function:

$$f(x) = \frac{\delta}{\sqrt{2\pi}} \quad \frac{\lambda}{x(\lambda - x)} \quad \exp \left\{ -\frac{1}{2}\left[\gamma + \delta \ln\left(\frac{x}{\lambda - x}\right)\right]^2 \right\}$$

$$0 < x < \lambda\ , \quad \delta > 0\ , \quad -\infty < \gamma > \infty\ , \quad \lambda > 0$$

$$\text{where} \quad \gamma + \delta \ln\left(\frac{x}{\lambda - x}\right) = z_x \approx N(0,1)$$

λ = scale parameter
γ, δ = shape parameters

Likelihood function:

$$L = \frac{\delta^n}{(2\pi)^{n/2}} \quad \frac{\lambda^n}{\prod_{i=1}^{n}\left(\lambda x_i - x_1^2\right)} \quad \exp \sum_{i=1}^{n}\left\{ -\frac{1}{2}\left[\gamma + d \ln\left(\frac{x_i}{\lambda - x_i}\right)\right]^2 \right\}$$

Loglikelihood function:

$$\text{Log}\,(L) = n \log \delta - \frac{n}{2}\log\,(2\pi) + n \log \lambda$$

$$-\sum_{i=1}^{n} \log\,(\lambda x_i - x_i^2) - \frac{1}{2}\sum_{i=1}^{n}\left[\gamma + \delta \ln\left(\frac{x_i}{\lambda - x_i}\right)\right]^2$$

Lognormal distribution

Probability density function:

$$f(x) = \frac{1}{(x-a)\, c\, (2\pi)^{1/2}} \quad \exp \left\{ \frac{-\left[\left(\log (x-a) - b\right)\right]^{2}}{2c^{2}} \right\}$$

$$0 \le x < \infty \quad , \quad 0 \le a \le x \quad , \quad b, c > 0$$

a = location parameter
b = scale parameter
c = shape parameter

Likelihood function:

$$L = \frac{1}{\prod_{i=1}^{n} \left(x_i - a\right) c^{n} \, (2\pi)^{n/2}} \quad \sum_{i=1}^{n} \exp \left[-\frac{1}{2} \left\{ \frac{\log (x_i - a) - b}{c} \right\}^{2} \right]$$

Loglikelihood function:

$$\operatorname{Log}(L) = - \sum_{i=1}^{n} \log (x_i - a) \quad - \quad \frac{n}{2} \log (2\pi) \quad - \quad n \log c$$

$$- \quad \frac{1}{2} \sum_{i=1}^{n} \left\{ \frac{\log (x_i - a) - b}{c} \right\}^{2}$$

Generalised normal distribution

Probability density function:

$$f(x) = (2\pi)^{1/2} \quad b^{-1} \exp \left(ky - y^{2} / 2\right)$$

$$\text{where } y = \begin{cases} -k^{-1} \ln \{1 - k(x-a)/b\} & , \quad k \neq 0 \\ (x-a)/b & , \quad k = 0 \end{cases}$$

and $a + b/k \leq x < \infty$ if $k < 0$

$-\infty < x < \infty$ if $k = 0$

$-\infty < x \leq a + b/k$ if $k > 0$

a = location parameter

b = scale parameter

Likelihood function:

$$L = (2\pi)^{-n/2} \quad b^{-n} \quad \exp \quad \sum_{i=1}^{n} (ky_i - y_i^2/2)$$

Loglikelihood function:

$$\text{Log}(L) = -\frac{n}{2}\log(2\pi) - n \log b + \sum_{i=1}^{n} (ky_i - y_i^2/2)$$

The details of the prediction of parameters of the distributions were described by Kamziah (1998).

Skewness and kurtosis

The skewness coefficient, $\sqrt{\beta_1}$, and kurtosis coefficient, β_2, of various statistical distribution measure the flexibility of the distribution in regard to their changes in shape.
Here

$$\sqrt{\beta_1} = \mu_3 / \mu_2^{3/2} \quad , \quad \beta_2 = \mu_4 / \mu_2^2$$

where $$\mu_k = \int_{-\infty}^{\infty} [x - E(x)]^k \, f(x) \, dx$$

and f(x) is the probability density function of the random variable x. Skewness, or symmetry, is defined as a departure from symmetry about the mean where negative values indicate a distribution with a long tail to the left and positive values a long tail to the right. Kurtosis is generally considered to be a relative measure of the flatness or peakness of a distribution; the larger the value of β_2, the more peaked is the distribution. The value of β_1 or β_2 does not itself uniquely define a distribution. It is helpful in identifying distributions that should not be fit.

The β_1–β_2 space is used to demonstrate the range of skewness and kurtosis covered by various statistical distribution (Johnson & Kotz 1970). The graph provides great information in considering the strengths and weaknesses of the distribution. Figure 1 presents the b_1–b_2 (the moment estimators of β_1–β_2) space for statistical distributions that have been suggested for describing diameter distribution. The 'impossible region' in the graph indicates that the combinations of β_1 and β_2 are mathematically impossible. By tradition we choose to present the ordinate scale upside down. The b_1–b_2 space presented simply spans the segment of the space appropriate to our discussion. Such a graph could suggest which distribution fits a set of data based on sample estimates of β_1 and β_2.

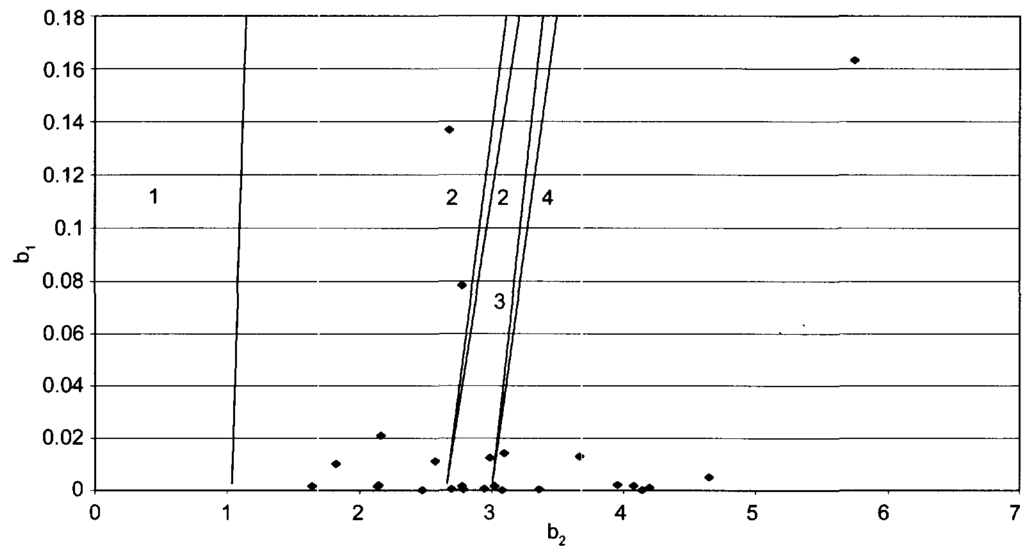

Figure 1. The location of the estimates b_1 and b_2 for the diameter measurements from the 25 data sets of Table 1. (1 : impossible region, 2 : Weibull distribution line, 3 : gamma distribution line, 4 : lognormal distribution line)

The gamma, lognormal and Weibull are represented by lines in the b_1–b_2 space, demonstrating their capability to assume a variety of shapes. Hosking (1986) verified that the lognormal distribution is a special case of the generalised normal distribution. The generalised normal includes both log normal distributions with positive skewness and a lower bound ($k<0$), and lognormal distributions with negative skewness and an upper bound ($k > 0$). It also includes the normal distribution as a special case ($k = 0$).

These lines fall rather close to each other; hence, it explains why sets of data can often be fitted equally well or equally poorly by either of these distributions. Their locations also explain why the Weibull distribution has often been found to give a "better fit" to diameter data than either the gamma or lognormal. A further distinction between these three distributions is their ability to represent different types of skewness. The lognormal and gamma distributions are limited to shapes that have positive skewness, while the Weibull distribution has the ability to describe both positive and negative skewness.

The Johnson S_B distribution (Johnson 1949) span the β_1–β_2 space based on transformation of a standard normal variate. The distribution covers the region above the lognormal line; therefore, it provides more flexibility in skewness and kurtosis (Hafley & Schreuder 1977).

The moment estimators of $\sqrt{\beta_1}$ and β_2 are

$$\sqrt{b_1} = \frac{\sum (X_i - \bar{x})^3}{\left[\sum (X_i - \bar{x})^2\right]^{3/2}}$$

and

$$b_2 = \frac{\sum (X_i - \bar{x})^4}{\left[\sum (X_i - \bar{x})^2\right]^2}$$

Application

We compare the five distributions in terms of how well they fit diameter data obtained from 16 uneven-aged stands of mixed species located at Bukit Lagong Forest Research, Kepong, Selangor, and 9 even-aged stands of *Acacia mangium* species located at Kemasul Forest Reserve, Pahang. The stands were all plantations and the ages ranged from 2 to 22 y. Table 1 shows the summary of the stand data.

A summary of the quality of fit based on the log of the likelihood for each data set is presented in Table 2. The log of the likelihood is used because it is much easier to compute than the likelihood and it provides the same results for ranking purposes. The purpose of the ranking is to identify one or more distributions which would perform well over a variety of empirical data sets (Hafley & Schreuder 1977).

Results

Table 1 presents estimates of $\sqrt{\beta_1}$ and β_2 and descriptive information regarding the data sets. About 37% of the diameter distributions are negatively skewed. The range of the skewness is from - 0.114 to 0.404 and the kurtosis ranges from 1.64

to 5.75. This phenomenon commonly results from differentiation of stands. This suggests that the distribution of the diameter is far from normal.

Table 1. Species composition, age, number of trees per plot and $\sqrt{b_1} + b_2$ for diameter distribution for each stand

Stand	Species*	Age	No. trees per plot	Min. diam. (cm)	Max. diam. (cm)	Mean diam. (cm)	($\sqrt{b_1}$)	(b_2)
1	SM	15	15	25.6	38.6	31.57	0.034	2.138
2	SL	15	20	14.6	24.6	20.12	-0.114	3.667
3	SL	19	30	23.0	37.5	30.76	-0.041	1.640
4	SMF	12	25	3.7	8.4	5.56	0.145	2.160
5	SWM	19	20	20.4	41.3	28.75	0.112	2.986
6	HS	18	20	12.0	22.0	16.76	0.370	2.678
7	SL	22	64	7.5	39.5	24.96	-0.071	4.645
8	MF	13	40	4.6	27.1	14.17	0.045	2.142
9	MU	15	20	4.4	11.9	7.91	0.040	2.775
10	SB	14	28	8.8	14.6	11.68	0.021	2.695
11	SBT	19	20	14.5	32.3	21.97	0.119	3.093
12	SR	17	20	10.2	24.5	17.23	-0.280	2.778
13	AS	14	20	11.5	25.3	17.30	0.106	2.570
14	SC	11	15	16.1	22.2	18.79	0.101	1.820
15	KM	18	15	7.4	14.1	9.31	0.404	5.748
16	DC	18	15	8.1	29.5	18.69	0.008	4.139
17	AM	5	266	6.0	22.8	16.48	-0.046	3.955
18	AM	6	286	6.2	23.6	18.12	-0.033	4.197
19	AM	7	259	7.3	26.5	17.78	-0.024	2.943
20	AM	8	251	6.0	20.0	20.04	-0.040	3.020
21	AM	10	431	4.0	30.4	19.69	-0.019	2.782
22	AM	9	284	7.4	30.5	20.52	-0.042	4.077
23	AM	10	118	13.9	34.6	23.94	0.019	3.361
24	AM	9	175	18.7	29.2	23.43	0.006	3.078
25	AM	8	154	16.9	28.0	22.76	0.004	2.756

*SM : *Shorea macroptera*, SL : *Shorea leprosula*, , SMF : *Shorea multiflora*, SWM : *Swietenia macrophylla*, HS : *Hopea sangal*, MF : *Mesua ferrea*, MU : *Madhuca utilis*, SB : *Scorodocarpus borneensis*, SBT : *Shorea bracteolata*, SR : *Shorea resinosa*, AS : *Anisoptera scaphula*, SC : *Shorea curtisii*, KM : *Koompassia malaccensis*, DC : *Dyera costulata*, AM : *Acacia mangium*.

In fitting the Johnson S_B distribution, we assume the lower bound $\varepsilon = 0$. We apply the maximum likelihood iteration to locate the upper bounds of Johnson S_B distribution. The specified lower bound is considered realistic value and appropriate to the ultimate use of the Johnson S_B distribution without substantially affecting the conclusion of solving the lower bound iteratively.

Table 2 shows the relative ranking of fitting statistical distributions to diameter data. The numbers in parentheses beside the loglikelihood values show the relative ranking of the distributions for the data sets. The rank sum for the 25 data sets for each statistical distribution is presented at the bottom of Table 2. The results show that Johnson S_B is the most consistent performer. It is the best distribution to fit in all but three instances. The Weibull is the second best fitting distribution. The lognormal is generally the third best distribution to fit the data sets. The generalised normal and gamma distributions are inferior to the Johnson S_B, Weibull and lognormal distributions in terms of their performance over the variety of stands represented.

Table 2. Log criterion and ranking (in parentheses) of the gamma (GM), generalised normal (GN), Johnson SB (JSB), lognormal (LN), and Weibull (WB) distributions for diameters with the rank sum of the relative rankings for the data sets

Stand	GM	GN	JSB	LN	WB
1	-32.18 (5)	-28.20 (4)	-19.60 (1)	-25.13 (2)	-26.15 (3)
2	-46.07 (5)	-33.04 (3)	-18.70 (1)	-34.49 (4)	-32.19 (2)
3	-66.76 (5)	-56.84 (4)	-39.50 (1)	-43.32 (2)	-55.68 (3)
4	-63.26 (5)	-38.07 (4)	-31.20 (1)	-33.97 (3)	-32.55 (2)
5	-48.76 (5)	-41.93 (4)	-30.50 (1)	-32.91 (2)	-38.56 (3)
6	-40.63 (5)	-35.25 (4)	-20.12 (1)	-27.40 (2)	-31.66 (3)
7	-172.96 (5)	-413.60 (4)	-88.00 (1)	-129.80 (3)	-129.48 (2)
8	-85.77 (4)	-87.34 (5)	-65.80 (2)	-60.90 (1)	-77.92 (3)
9	-43.88 (5)	-32.93 (4)	-17.80 (1)	-28.10 (2)	-29.61 (3)
10	-60.00 (5)	-41.72 (4)	-36.80 (1)	-39.70 (3)	-37.36 (2)
11	-45.35 (5)	-38.46 (4)	-25.10 (1)	-33.20 (2)	-36.04 (4)
12	-41.55 (5)	-38.64 (4)	-22.90 (1)	-31.90 (2)	-35.18 (3)
13	-41.98 (5)	-36.45 (4)	-27.20 (1)	-30.40 (2)	-34.58 (3)
14	-26.30 (5)	-23.52 (4)	-17.20 (1)	-18.20 (2)	-21.69 (3)
15	-39.03 (5)	-23.21 (4)	-12.30 (1)	-19.10 (2)	-20.32 (3)
16	-32.72 (5)	-29.82 (4)	-19.20 (1)	-23.60 (2)	-28.07 (3)
17	-1092.55 (5)	-465.34 (2)	-307.70 (1)	-531.48 (4)	-485.52 (3)
18	-879.04 (5)	-528.60 (2)	-328.95 (1)	-565.15 (3)	-639.85 (4)
19	-973.97 (5)	-512.45 (3)	-361.39 (1)	-889.23 (4)	-500.56 (2)
20	-1042.88 (5)	-423.93 (1)	-696.00 (4)	-481.52 (2)	-492.82 (3)
21	-1139.25 (5)	-750.97 (2)	-643.78 (1)	-1145.11 (5)	-990.78 (3)
22	-638.28 (4)	-494.30 (2)	-365.47 (1)	-807.35 (5)	546.04 (3)
23	-363.02 (5)	-222.99 (2)	-232.68 (3)	-304.15 (4)	-216.53 (1)
24	-596.88 (5)	-314.30 (3)	-217.99 (1)	-321.56 (4)	-284.01 (2)
25	-337.58 (5)	-276.41 (3)	-168.90 (1)	-283.22 (4)	-250.75 (2)
Rank sum	122	84	31	71	67

Figure 1 shows that the observations appear to be consistent with the implications on the plot of the b_1 and b_2 points. Most of the points are not close to the lines associated with the Weibull, gamma, and lognormal distributions. The sign of b_1 is inappropriate to those points which fall close to these lines. Regardless of the sign of $\sqrt{b_1}$, the gamma distribution is always the poorest performing distribution. However, the calculation of b_1 and b_2 is insufficient for selecting the best distribution for a given data set. This is obvious as since the region of the β_1 and β_2 space spanned by the S_B distribution and the lognormal, gamma and Weibull lines overlap, calculated values of b_1 and b_2 will not identify which of these two distributions will give better fit to a given data set.

Conclusion

The relative ranking of the data set shows that each of the statistical distributions considered for fitting a variety of empirical data sets has strengths and weaknesses. In this study the Johnson S_B distribution demonstrates a relative stability across the data sets. Flexibility of the Johnson S_B distribution is considered in terms of its ability to fit empirical data sets and relative simplicity to apply through the method of maximum likelihood estimator even though the upper bound of the data set is not known.

The statistical procedures used to describe the diameter distribution can further be applied for the development of growth and yield models. The estimation of parameters based on diameter will lead to prediction of parameters of the distribution related to stand characteristics such as age, average height, total basal area per hectare and number of trees per unit area by developing regression equations. Hence, a growth model could assist forest researchers and managers to examine the alternative cutting limits and likely outcomes, and thus to make their decisions objectively.

References

BAILEY, R. I. & DELL, T. R. 1973. Quantifying diameter distributions with the Weibull functions. *Forest Science* 19(2):97–104.

BALDWIN, V. C., JR., & FEDUCCIA, D. P. 1987. *Loblolly Pine Growth and Yield Prediction for Managed West Gulf Plantations.* USDA Forest Service Research Paper SO - 236. 27 pp.

BLISS, C. I. & REINKER, K. A. 1964. A lognormal approach to diameter distribution in even-aged stands. *Forest Science* 10:350–360.

BROOKS, J. R., BORDERS, B. E. & BAILEY, R. L. 1992. Predicting diameter distributions for site-prepared loblolly and slash pine plantations. *Southern Journal of Applied Forestry* 16:130–133.

CLUTTER, J. L. & ALLISON, B. J. 1974. A growth and yield model for *Pinus radiata* in New Zealand. Pp. 136–160 in *Growth Models for Tree and Stand Simulation.* Department of Forest Yield Research. Note 30. Royal College of Forestry, Stockholm, Sweden.

CLUTTER, J. L. & BENNETT, F. A. 1965. *Diameter Distributions in Old-Field Slash Pine Plantations.* Georgia Forest Research Council Report 13. 9 pp.

FEDUCCIA, D. P., DELL, T. R., MANN, W. F., CAMPBELL, T. E. & Polmer, B. H. 1979. *Yields of Unthinned Loblolly Pine Plantations on Cutover Sites in the West Gulf Region.* USDA Forest Service Research Paper SO-148. 88 pp.

HAFLEY, W. L & SCHREUDER, H. T. 1977. Statistical distributions for fitting diameter and height data in even-aged stands. *Canadian Journal of Forest Research* 7:481–487.

HAFLEY, W. L., SMITH, W. D., & BUFORD, M. A. 1982. *A New Yield Predictions Model for Unthinned Loblolly Pine Plantations.* Southern Forest Research Centre, School of Forest Research, North Carolina State University Technical Report 1. 65 pp.

HOSKING, J. R. M. 1986. *The Theory of Probability Weighted Moments.* IBM T. J. Watson Research Centre, New York, USA. : 77–79.

KAMZIAH, A. K. 1998. Development of Diameter Distribution Yield Prediction Models for Simulation of *Acacia mangium* Plantations. M.S. thesis, Universiti Putra Malaysia, Serdang, Malaysia.

JOHNSON, N. L. 1949. Systems of frequency curves generated by methods of translation. *Biometria* 36:149–176.

JOHNSON, N. L. & KOTZ, S. 1970. *Distributions in Statistics.* Houghton Mifflin Co., Boston MA: 9–27.

LENHART, D. J. & CLUTTER, J. L. 1971. *Cubic-foot Yield Tables for Old-Field Loblolly Pine Plantations in the Georgia Piedmont.* Georgia Forest Research Council Report 22, Series 3. 13 pp.

MATNEY, T. G., & Sullivan, A. D. 1982. Compatible stand and stock tables for thinned loblolly pine stands. *Forest Science* 28:161–171.

MCGEE, C. E. & DELLA-BIANCA, L. 1967. *Diameter Distributions in Natural Yellow-Poplar Stands.* USDA Forest Service Research Paper SE-25. Southeast Forest Experimental Station, Asheville. 7 pp.

MEYER, H. A. & STEVENSON, D. D. 1943. The structure and growth of virgin beech-birch-maple-hemlock forests in northern Pennsylvania. *Journal of Agricultural Research* 67:465–484.

MEYER, W. H. 1930. *Diameter Distributions Series in Even-Aged Forest Stands.* Yale University School of Forestry Bulletin 28. 205 pp.

NELSON, T. C. 1964. Diameter distribution and growth of loblolly pine. *Forest Science* 10:105–115.

OSBORNE, J. G. & SCHUMACHER, F. X. 1935. The construction of normal-yield and stand tables for even-aged timber stands. *Journal of Agricultural Research* 51: 547–564.

SMALLEY, G. W. & BAILEY, R. L. 1974. *Yield Tables and Stand Structure for Loblolly Pine Plantations in Tennessee, Alabama, and Georgia Highlands.* USDA Forest Service Research Paper SO-96. 81 pp.

ZOHRER, F. 1972. The beta distribution for best fit of stem-diameter-distribution. Pp. 91–106 in *3rd Conference Advisory Group of Forest Statistics Proceedings, IUFRO.* Institute National Recherche Agronomique, Paris, Publ. 72.3.