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NEW GROWTH MODELS FOR *CUPRESSUS LUSITANICA* AND *PINUS PATULA* IN KENYA

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NGUGI, M. R., MASON, E. G. & WHYTE, A. G. D. 2000. New growth models for *Cupressus lusitanica* and *Pinus patula* in Kenya. Stand growth and yield models for *Cupressus lusitanica* and *Pinus patula* grown in Kenya were revised, and growth trajectories in Uasin Gishu, Elburgon, Njoro, Kiambu, Nyeri and Mt. Kenya regions were identified. Permanent sample plot data collected between 1964 and 1985 were used for the analyses. Growth functions in difference equation form were fitted to mean top height and basal area data with dummy variables used to segregate regions in a single equation. Modified forms of the Schumacher polymorphic equation gave the best fit for both variables, while the use of thinning indices provided much improvement to the fit of the basal area equation. It was found that *C. lusitanica* in Elburgon and Mt. Kenya has a different growth pattern from the other regions. Best sites for *P. patula* basal area and height growth were identified as Kiambu and Nyeri respectively. Accurate projections of stand growth were realised for each region.

Key words: *Cupressus lusitanica* - *Pinus patula* - Kenya - stand growth model - difference equation - dummy variables

NGUGI, M. R., MASON, E. G. & WHYTE, A. G. D. 2000. Model pertumbuhan baharu bagi *Cupressus lusitanica dan Pinus patula* di Kenya. Model pertumbuhan dirian dan model hasil bagi *Cupressus lusitanica* dan *Pinus patula* yang ditanam di Kenya disemak, dan trajektori pertumbuhan di kawasan Uasin Gishu, Elburgon, Njoro, Kiambu, Nyeri dan Gunung Kenya dikenal pasti. Data daripada petak sampel kekal yang diambil antara tahun 1964 hingga 1985 digunakan untuk dianalisis. Fungsi pertumbuhan dalam bentuk persamaan yang berbeza dipadankan dengan min ketinggian dominan dan data luas pangkal dengan pembolehubah patung digunakan untuk mengasingkan kawasan-kawasan dalam persamaan individu. Bentuk-bentuk yang diubahsuai bagi persamaan polimorf Schumacher merupakan padanan terbaik bagi kedua-dua pembolehubah, manakala penggunaan indeks penjarangan memberikan peningkatan bagi memadankan dengan persamaan luas pangkal. Didapati *C. lusitanica* di Elburgon dan Gunung Kenya mempunyai pola pertumbuhan yang berbeza daripada kawasan lain. Tapak terbaik bagi luas pangkal dan pertumbuhan ketinggian *Pinus patula* dikenal pasti masing-masing sebagai Kiambu dan Nyeri. Unjuran yang tepat bagi pertumbuhan dirian direalisasikan bagi setiap kawasan.

Introduction

The government of Kenya currently owns over 160 000 ha of exotic plantations across the country. *Cupressus lusitanica* (cypress) and *Pinus patula* (pine) form the bulk of

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these plantations with proportions of 45 and 26 percent respectively (MENR-FINNIDA 1994). These stands are managed for the production of sawlog, pulpwood and plywood, and are the sole source of raw materials for the local wood processing industries with a supply of 1.4 million $m^3 y^{-1}$ (Sean 1997).

The plantations are sparsely located in the highland areas, and forest blocks within each region supply the wood demand of the adjacent industries. This is particularly so because of the long distances and poor infrastructure between the regions. Establishment of permanent sample plots started in 1964 to monitor the growth of the first rotation crop and provide data for yield prediction. Some of the data collected until 1974 have been previously used for subsequent yield analyses (Wanene & Wachiori 1975, Alder 1977, Mathu & Philip 1979). However, more data for the later stages of stand growth were collected until 1985, necessitating development of more comprehensive growth and yield models. Tennent (1990) developed growth models from the entire data set and fitted single multivariate equations for each species for the whole of Kenya. The equations were characterised by poor fit for the data set and errors associated with the use of derived variables as predictor variables were found to be propagated as projections were made using the equations. Further, these equations failed to characterise growth differences between regions. As a result, yields of some regions have been found to be overestimated and others underestimated. The consequences of this problem have been most critical when establishing sustainable levels of wood supply and preparation of felling plans (Ngugi 1998).

In this paper, new models are presented for projecting stand basal area and mean top height of even-aged forest plantations. A range of sigmoid functions is investigated and dummy variables are used to characterise locations. The accuracy of these models is tested using bias, residual mean square and mean of absolute residuals.

Methods

A difference equation method (Clutter *et al.* 1983) was used. This method is a generalisation of simultaneous growth and yield models by Sullivan and Clutter (1972). Pairs of consecutive measurements of each variable to be predicted (height and basal area) are used such that the first measurement at a specific time is used to predict the expected status at the second measurement. For example, basal area (G_2) at time (T_2) is predicted as a function of initial basal area (G_1) and the assessment age (T_1).

A variety of sigmoid-shaped functions that have been successfully adopted to model growth and yield of forest stands through time were used. Anamorphic and polymorphic forms of Schumacher log-reciprocal equation (Clutter & Jones 1980), Hossfeld (Woollons *et al.* 1990), Chapman-Richards (Pienaar & Turnbull 1973), and Gompertz (Nakoe 1978) equations were fitted to the mean top height and basal area data. The classification of equations into anamorphic and polymorphic forms is primarily based on the shape of the family of curves generated (Borders *et al.* 1984). Anamorphic curves are generated by fixing the slope parameter, yielding curves having a common slope but different asymptotes while polymorphic curves are generated by fixing the asymptote, resulting in curves with different shapes but a common asymptote (Aurelio *et al.* 1992, Vanclay 1994). Though there are no particular advantages in using either of these two forms of equations when fitting a model, studies have shown that variability in treatments, competition, responses of a crop and site factors are usually much better represented by polymorphic equations (Mason & Whyte 1997). The equation formulations used are listed as follows:

1. Hossfeld polymorphic form:

$$Y_2 = \frac{1}{\frac{1}{Y_1}[(\frac{T_1}{T_2})^{\gamma} + \frac{1}{\alpha}(1 - (\frac{T_1}{T_2})^{\gamma})]} \tag{1}$$

2. Hossfeld anamorphic form:

$$Y_2 = \frac{1}{\frac{1}{Y_1} + \theta(\frac{1}{T_2^{\beta}} - \frac{1}{T_1^{\beta}})} \tag{2}$$

3. Schumacher polymorphic form:

$$Y_2 = Y_1^{(T_1/T_2)^{\gamma}} \exp \alpha(1 - (\frac{T_1}{T_2})^{\gamma}) \tag{3}$$

4. Schumacher anamorphic form:

$$Y_2 = Y_1 \exp \beta(\frac{1}{T_1^{\alpha}} - \frac{1}{T_2^{\alpha}}) \tag{4}$$

5. Chapman- Richards anamorphic form:

$$Y_2 = \frac{Y_1[1 - \exp(-\beta T_1)]}{[1 - \exp(-\beta T_2)]^{1/(1-\gamma)}} \tag{5}$$

6. Gompertz polymorphic form:

$$Y_2 = \exp(\log(Y_1)\exp[-\gamma(T_2 - T_1) + \delta(T_2^2 - T_1)] + \alpha(1 - \exp[-\gamma(T_2 - T_1) + \delta(T_2^2 - T_1^2)]))$$
6)

where β, δ, α, γ and θ are parameters to be estimated, and Y_1 = yield at age T_1 and Y_2 = yield at age T_2 are the variables of interest.

Site variations

In most countries forests grow on vast areas, with great potential variability with respect to soil type, rainfall, altitude, aspect, stocking, genotype, species and cultural treatment. Nevertheless, general growth projection systems that cover large forest areas have been developed to provide information for forest management plans preparation, timber inventory assessments and policy making among other reasons.

However, when intensive forest management demands growth prediction sensitive at regional or sub-regional levels, general growth models lose their credibility (Whyte *et al.* 1992).

Several approaches have been used to localise growth models. The method of stratifying a large population into smaller homogeneous sub-populations was used successfully to fit height equations for radiata pine grown in New Zealand, resulting in polymorphic curves (Burkhart & Tennent 1977). Leary and Hamlin (1987) argued that calibration of an existing growth model to cater for local conditions does not just happen, but requires additional data or even fitting completely new models which are more flexible for future calibrations. Smith (1983) used the double sampling technique to calculate an annual adjustment factor of diameter increments of the STEMS (Belcher *et al.* 1983) model. The empirical Bayes method of estimating model coefficients has also been successfully used (Green *et al.* 1992) in simultaneously estimating Honduran pine yield equation coefficients for 21 different soil sites.

The use of dummy variables 1 and 0 as an alternative technique to the standard analysis of variance and covariance (Gujarat 1970) has been widely used. This method allows the formulation of an analysis of covariance among regions or data sets, by representing each as a dummy variable within a single equation. The coefficients that are different from zero at the 0.05 significance level are retained. The method was successfully applied by Monserud (1984) in Douglas fir site index equations for different habitat types, Ferguson (1979) to distinguish growth coefficients of four radiata pine forests in Australian Capital Territory, Whyte *et al.* (1992) to model growth of Douglas fir in the whole of South Island of New Zealand, and Mason (1992) for radiata pine initial growth models in New Zealand. This method was used in this study to localise the component equations.

Materials

Exotic plantations in Kenya are managed under two treatment regimes, sawlog and pulpwood. The initial spacing and stocking for sawlog regime are 2.5 × 2.5 m and 1600 stems ha^{-1} while for pulpwood they are 2.75 × 2.75 m and 1322 stems ha^{-1} respectively. Sawlog regime has four thinnings, with the final harvesting at age of 30 y. Harvesting for the pulpwood regime is scheduled at the age of 15 y with the option of a thinning at the same age if harvesting is delayed, and a final harvest should not exceed 20 y. The data used in the present study were obtained from the Forest Inventory permanent sample plots database for even-aged forest plantations. Establishment of the plots commenced in 1964, with more plots being added annually on new-planted sites. The age of forest stands at the time of plot location was in the range of 5.5 to 34 y. A plot area of 0.04 ha was used and maintained throughout, with measurements of diameter at breast height (1.3 m) of every tree and top height of four dominant trees collected annually using diameter tape and Suunto clinometer respectively. The number of annual measurements per plot was between 3 and 16, with 42 and 35 y being the age of the oldest stands of cypress and pine respectively, at the time of the last data collection in 1985. Unfortunately no more data on permanent sample plot have been collected since. On several occasions where annual measurement could not be done because of logistic problems, data on thinned plots were often lost.

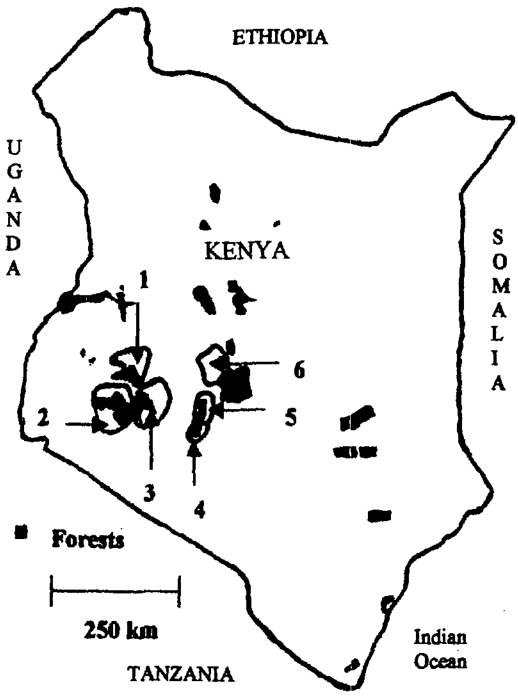

Figure 1. Map of Kenya showing the locations of plantation forests and each of the six regions

Data from each plot were summarised into mean top height (average height of the 100 trees per hectare with largest diameter at breast height and is often referred to as the dominant height) and net basal area per hectare (basal area of a fully stocked site) for every measured year. The data comprised 1434 and 1714 annual measurements of cypress and patula pine, from 327 plots distributed in six forest growing regions, Uasin Gishu, Elburgon, Njoro, Kiambu, Nyeri and Mt. Kenya (Figure 1). The regions were grouped into wet (Uasin Gishu and Kiambu), medium (Elburgon and Nyeri) and dry (Mt. Kenya and Njoro), based on rainfall amounts, and the number of plots established was relative to the size of planted forests (Table 1). Site index value for each plot was calculated from the mean top height equation developed in this study, at a site index reference age of 15 y, and altitude values were derived from a topographic map.

Validation data set

Setting aside a comprehensive data set for the purpose of validation was not possible because segregation of data into regions resulted in limited data sets for fitting the respective models. However, data from nine plots were set aside, with at least one plot selected randomly from each region for use in validating the models developed. The selected plots for cypress were plots No. 233 located in Mt. Kenya (Gathiuru forest),

Table 1. Location, number of permanent sample plots per region and the number of annual measurements

Classification	Region	*Cupressus lusitanica* No. of plots	*Cupressus lusitanica* No. of annual measurements	*Pinus patula* No. of plots	*Pinus patula* No. of annual measurements
Wet	Uasin Gishu	46	448	56	448
	Kiambu	28	247	28	266
Medium	Elburgon West	32	280	33	347
	Nyeri	40	237	22	283
Dry	Mt. Kenya	6	149	16	176
	Njoro	6	73	14	194
Total		158	1434	169	1714

No. 190 located in Elburgon (Kitiro forest) and No. 35 in Uasin Gishu (Buret forest). For patula pine model, the plots were plots No. 1 and 10 in Uasin Gishu (Buret forest and Timboroa, respectively), No. 215 in Mt. Kenya (Nanyuki forest), No. 241 in Elburgon (Sokoro forest), No. 268 in Kiambu (Kinale forest) and No. 312 in Nyeri (Kabage forest). The observed values in these plots for each particular region were compared graphically with those predicted by the new models with respective adjustments using dummy variables and thinning indices incorporated.

Analysis and results

Derivative free algorithm method (Dud) for nonlinear least squares (Raltson & Jennrich 1978) was applied using SAS statistical software. To avoid proliferation of growth equations by fitting each region separately, residual errors resulting from fitting each of these equations to the entire data set were analysed using plot, chart and univariate procedures of SAS software. Graphical plots of residuals against predictor variables and predicted values, classified into regions, were used to examine independent distributions (unbiased), systematic deviations (biased) and outlier data. In a few cases the Jackknife method was used to isolate specific outliers. Residual frequency charts were used to examine the shape of distributions and provided inferences on skewness and apparent departures from normal distribution. Skewness, kurtosis, mean residual, and normal probability plot among other statistics generated from the univariate procedure were used to supplement inferences from residual patterns and for testing normality assumption that residual errors were normally and independently distributed with mean zero and constant variance (Draper & Smith 1981). Small residual mean square (RMS) value was also used as an indication of a good fit for the data. The F-test and coefficient of determination values (r^2) were not used in the analysis because of autocorrelation inherent in data obtained from repeated measurements of the same sets of trees. This has been found to result in underestimation of the error terms (Hildebrand & Ott 1991). The best fitting equation was chosen for further modification using additional predictor variable.

Localisation of the best fitting equation by the use of dummy variables was attained by assigning one region as the default and every other region was represented by a separate coefficient linearly related to the default coefficient (α). This resulted in a small number of plots for individual species in the specific region. Hence, the approach of using the whole data set and utilising residual analysis as indicators of the goodness of fit (Whyte *et al.* 1992) was adopted and complemented by a highly limited set of data for the purpose of validation. The strength of this approach was that bigger and more representative data set used to fit regional equations provided better coefficient estimates. In the equation formulation the regions were represented by dummy variables *k*1, *k*2, *k*3, *k*4 and k5, and coefficient estimates β_1, β_2, β_3, β_4, β_5 respectively. Upon fitting the model, the estimated coefficients were scrutinised to confirm whether they were significantly different from zero. Regions whose coefficients were not significantly greater or less than zero were considered to exhibit similar growth trajectory to the default region. Consequently the data for such regions were combined and the analysis repeated to derive unbiased coefficients for the combined regions.

Cupressus lusitanica mean top height model

The estimated coefficients of the general equations fitted to the cypress data, their standard errors and residual mean square (RMS) values are shown in Table 2. Since the equation with the least biased residual plots was often found to have the lowest RMS value, this value was used to indicate the best fitting equation.

The Schumacher polymorphic equation (3) with RMS of 0.5814 was found to give a better fit than most other equations and a slightly better fit than the Hossfeld polymorphic equation. This implied that the height growth is a log-reciprocal function of age. The equation was, therefore, considered for further regional sensitivity tests by assigning Kiambu as the default region and each of the other five regions being represented by a unique variable. Initial statistical outputs indicated that the

Table 2. Coefficient estimates, standard error (in brackets) and residual mean square values (σ^2) for general growth equations fitted to cypress height data set

Function	Coefficient estimates					
	α	β	γ	δ	θ	σ^2
Hossfeld polymorphic	47.9206 (1.9831)	-	1.1751 (0.0377)	-	-	0.5850
Hossfeld anamorphic	-	1.6173 (0.0218)	-	-	1.6164 (0.0235)	0.7485
Schumacher polymorphic	4.3920 (0.0995)	-	0.4736 (0.0329)	-	-	0.5814
Schumacher anamorphic	-	5.0181 (0.1019)	0.4459 (0.0363)	-	-	0.6086
Chapman-Richards anamorphic	-	0.0504 (0.004)	1.9616 (0.0366)	-	-	0.6146
Gompertz polymorphic	3.6793 (0.0487)	-	0.0910 (0.0035)	0.0007 (0.0001)	-	0.5940

coefficients for Uasin Gishu, Nyeri and Njoro were not significantly different from zero. Hence, data for these regions and those of Kiambu were pooled together to form the default while Elburgon and Mt. Kenya were represented with dummy variables *k*1 and *k*2 respectively as presented in equation 7.

$$\hat{h}_2 = h_1^{(T_1/T_2)^\gamma} \exp[(\alpha + \beta_1 k1 + \beta_2 k2)(1 - (\frac{T_1}{T_2})^\gamma)] \quad (7)$$

where $\hat{h}_2$ = estimated height at age T_2, h_1 = measured height at age T_1. Coefficient estimates and their standard errors (in brackets) were

$\hat{\gamma}$ = 0.4749 (0.0327) $\hat{\alpha}$ = 4.3748 (0.0984)

$\hat{\beta}_1$ = 0.1671 (0.0549) $\hat{\beta}_2$ = -0.1867 (0.0683)

$\hat{\sigma}^2$ (RMS) = 0.5735

Using dummy variables reduced the error sum of squares by 1.4%. Though altitude (height above sea-level) has been found to be an important variable for explaining variations in height growth (Woollons & Hayward 1985, Whyte *et al.* 1992, Mason & Whyte 1997), it was not very reliable here because no accurate figures were available. Ground preparation and weed control have also been found to be linearly related to height growth (Mason & Whyte 1997), but such data were not available. The asymptote α in equation 7 is the default dummy variable representing the four combined regions. When projections are made for these four regions, *k*1 and *k*2 assume a value of 0; when projections are made for Elburgon, the asymptote is $\hat{\alpha} + \hat{\beta}_1 k1$ because k_2 assumes a value of 0. For Mt. Kenya, the asymptote was given by $\hat{\alpha} + \hat{\beta}_2 k2$. A normal probability plot of residuals expressed a gentle *S*-shaped curve, an indication that they were outlier-prone. Because extreme residuals had been identified using graphical scatter plots and the Jackknife method, it was not justifiable to exclude any other data as outliers. Moreover, residual patterns of mean top height plotted against predicted value (Figure 2) portrayed a uniform distribution of residuals without major bias,

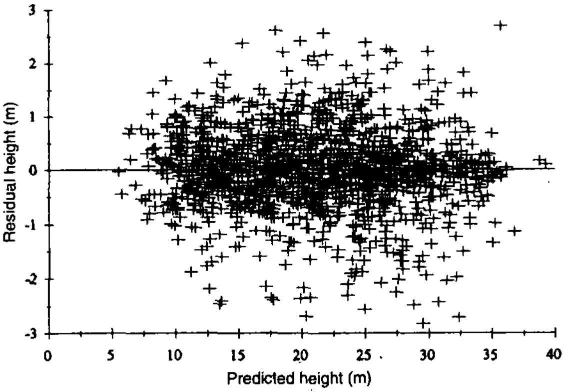

Figure 2. Residual plotted against predicted values (m) for cypress mean top height model

indicative of a good fit. The mean of absolute residuals for the equation of 0.55 m was found to be a 40% improvement compared with 0.91 m of Tennent's (1990) model.

These results clarify the former classification of Wanene and Wachiori (1975) by identifying the Elburgon region as site class I, Uasin Gishu, Kiambu, Nyeri and Njoro as site class II and Mt. Kenya as site class III. The results also confirm their finding that 92% of the cypress plantations are in site class II and agree with the findings of Muchiri (1992) that Kiambu is in site class II.

Cupressus lusitanica net basal area model

Comparison of the candidate equations fitted to cypress basal area data are presented in Table 3. The Schumacher polymorphic equation form was found to give a better fit and was chosen for further improvement using additional variables. Examination of extreme residuals greater than 10 m^2 and less than -10 m^2 showed that they coincided with thinning, but since only a few plots had not undergone thinning, it was not advisable to exclude any data. The approach which was utilised was that of including the entire data set and then using a thinning index at the point of the thinning (Bailey & Ware 1983, Murphy & Farrar 1988) to model the cause of the variation. Subsequent improvements of this equation to fit local adaptations was accomplished by introducing site index, altitude and dummy variables.

Thinning index

Two thinning indices Xt and Xa were employed as defined by Murphy and Farrar (1988):

$Xt = 1 - (Dt/Db)$	if $Dt/Db \neq 0$	silvicultural thinning
$= 0$	if $Dt/Db = 0$	no thinning
$Xa = (Da/Db) - 1$	if $T_2 - T_1 > 2$	delayed measure after thinning

Table 3. Coefficient estimates, standard error (in brackets) and residual mean square values (σ^2) for general growth equations fitted to cypress basal area data set

Function	Coefficient estimates					
	α	β	γ	δ	θ	σ^2
Hossfeld polymorphic	86.3437 (3.2086)	-	1.6504 (0.0484)	-	-	2.7283
Hossfeld anamorphic	-	2.0737 (0.0390)	-	-	2.0685 (0.0338)	5.2417
Schumacher polymorphic	4.8844 (0.0764)	-	0.7125 (0.0406)	-	-	2.6184
Schumacher anamorphic	-	6.8002 (0.1193)	0.4352 (0.0352)	-	-	2.9566
Chapman-Richards anamorphic	-	1.7693 (0.0287)	0.0407 (0.0041)	-	-	3.0549
Gompertz polymorphic	0.1145 (0.0055)	-	0.0013 (0.0001)	4.4498 (0.0483)	-	2.9645

where Dt = quadratic diameter thinned, Da = quadratic diameter after thinning, Db = quadratic diameter before thinning, Xt, Xa = thinning indices, T_1 = last measured age before thinning and T_2 = first measured age after thinning.

The thinning index Xt was used when basal areas before and after thinning were known. On a few plots the value of Xt was found to be less than zero. This reflected cases where big diameter trees of bad form were thinned and incidences of deviating from the recommended thinning practices where the bigger trees were thinned instead of small and suppressed trees (Muchiri 1992). A similar situation was encountered for Xa. The index Xa was used in cases where basal area before thinning was known and no measurements were taken immediately after thinning. The latter case was easily recognised in that there was a stocking reduction of above 150 stems ha^{-1} and the basal area measured two to three years after thinning was higher than the basal area before thinning.

Site index value for each plot and altitude values for each region were not included in the final equation formulation (9) because their coefficients were found not to be significant when used together with dummy variables. This indicated that there was little variability in site conditions within individual regions than between regions. The use of another set of dummy variables to represent pulpwood and sawlog regimes respectively was also not significant. This shows that there was hardly any noticeable difference between relative growth rates of stands generated through the two management regimes. The use of the thinning index Xt resulted in a 50% reduction in the residual mean square. A second thinning index Xa and default dummy variable (Kiambu, Uasin Gishu and Nyeri), $k1$ (Elburgon and Njoro) and $k2$ (Mt. Kenya) resulted in 15% more improvement.

$$\hat{G}_2 = G_1^{(T_1/T_2)^\gamma} \exp[(\alpha + \beta_1 k1 + \beta_2 k2)(1 - (\frac{T_1}{T_2})^\gamma)] + \beta_3 Xt(\frac{1}{T_2} - \frac{1}{T_1})\frac{Tt}{T_2} + \beta_4 Xa(\frac{1}{T_2} - \frac{1}{T_1})\frac{Tt}{T_2}] \quad (8)$$

where $\hat{G}_2$ = estimate of basal area at age T_2, G_1 = basal area at age T_1, Tt = thinning age, Xt, Xa = thinning indices. The coefficient estimates and their standard error (in brackets) were

$\hat{\alpha}$ = 4.8616 (0.0409) $\hat{\gamma}$ = 0.8615 (0.0277)

$\hat{\beta}_1$ = -0.0857 (0.0263) $\hat{\beta}_2$ = -0.3091 (0.0323)

$\hat{\beta}_3$ = 43.7847 (1.1836) $\hat{\beta}_4$ = 32.0688 (2.6306) and $\hat{\sigma}^2$ (RMS) = 1.117.

Uasin Gishu, Kiambu and Nyeri were grouped together and represented by the default dummy variable. The overall improvement narrowed the extreme range of residual of equation 8 from above ± 12 to a maximum residual of 4.4 m^2 ha^{-1} and minimum residual of -4.2 m^2 ha^{-1} as shown in Figure 3, with the residuals being well distributed without major bias. However, the normal probability plot portrayed a gentle *S*-shaped curve, an indication that the residuals were outlier prone. This

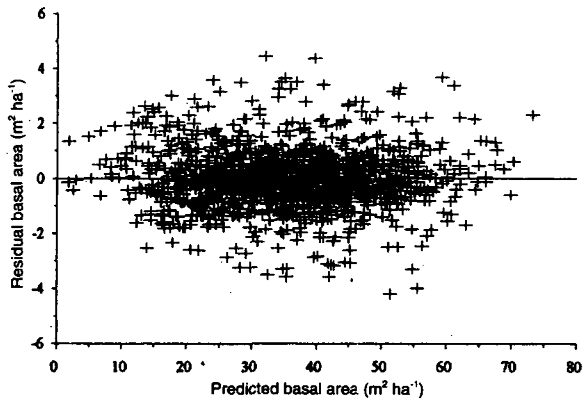

Figure 3. Residual plotted against predicted values (m^2 ha^{-1}) for cypress basal area ha^{-1} model

equation predicted basal area ha^{-1} with a mean of absolute residuals of 0.77 m^2 ha^{-1} which was a 50% improvement compared to 1.57 m^2 ha^{-1} of the Tennent (1990) model. The problem of irregular treatment of plantations, particularly the deviation from recommended thinning practices, has been cited as a major cause of the variability in basal area between stands of similar age (Muchiri 1992).

Pinus patula height model

The coefficients and the residual mean squares for the six candidate equations fitted to the pine height data set are presented in Table 4. The Schumacher polymorphic form with a residual mean square of 0.7182 was found to give the best fit to the data. Segregating the data into regions using dummy variables showed that there were no

Table 4. Coefficient estimates, standard error (in brackets) and residual mean square values (σ^2) for general growth equations fitted to patula pine height data set

Function	Coefficient estimates					
	α	β	γ	δ	θ	σ^2
Hossfeld polymorphic	1.5361 (0.0315)	-	41.0129 (0.7455)	-	-	0.7359
Hossfeld anamorphic	-	1.9141 (0.0237)	-	-	1.8961 (0.0201)	1.1294
Schumacher polymorphic	3.9685 (0.0345)	-	0.7725 (0.0275)	-	-	0.7182
Schumacher anamorphic	-	6.2234 (0.2037)	0.7091 (0.0302)	-	-	0.8289
Chapman-Richards anamorphic	-	-0.0946 (0.0038)	1.6716 (0.0201)	-	-	0.8333
Gompertz polymorphic	3.6884 (0.0332)	-	0.1431 (0.0042	0.0020 (0.0001)	-	0.7689

significant differences between coefficients for Uasin Gishu, Kiambu, Elburgon, Mt. Kenya and Njoro. It was found that only Nyeri expressed a different growth pattern from the rest of the regions as shown in equation 9.

$$\hat{h}_2 = h_1^{(T_1/T_2)^\gamma} \exp[(\alpha + \beta_1 k1)(1 - (\frac{T_1}{T_2})^\gamma)] \quad (9)$$

Coefficient estimates (and standard error) were

$\hat{\gamma} = 0.7985\ (0.0285)$ $\hat{\alpha} = 3.9250\ (0.0342)$

$\hat{\beta}_1 = 0.1178\ (0.0329)$ $\hat{\sigma}^2$ (RMS) = 0.7134.

The coefficient for Nyeri indicates a greater potential in height growth with trees growing 3 to 4 m taller by the time of harvest at age 30 y at any specified site index than the equivalent in other regions. The residual pattern of this equation (Figure 4) indicated no sign of bias and a mean of absolute residuals of 0.64 m indicated a 25% improvement compared to 0.85 m of the Tennent's previous equation. These results show that much of the variability expressed in height data is greater within regions than between regions. They contradict the generalised site classification by Wanene and Wachiori (1975) perhaps because the latter considered each stand measurement independent of location.

Pinus patula basal area model

The Schumacher polymorphic function gave the best fit (Table 5) for the data and was chosen for further improvement by incorporating thinning indices, site index, altitude and dummy variables. The problem of delay in re-measurements after thinning operation had taken place was more serious in the pine data set compared to cypress. However, the use of thinning indices *Xt* and *Xa* together with dummy variables reduced the residual mean square of the basic equation form by 50%. Site index alone provided a slight improvement of 0.96%, but its coefficient was within

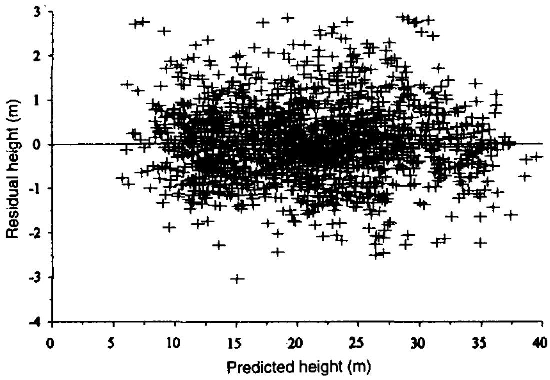

Figure 4. Residual plotted against predicted values (m) for patula pine mean top height model

Table 5. Coefficient estimates, standard error (in brackets) and residual mean square values (σ^2) for general growth equations fitted to patula pine basal area data set

Function	Coefficient estimates					
	α	β	γ	δ	θ	σ^2
Hossfeld polymorphic	55.3031 (0.7997)	-	2.0741 (0.0476)	-	-	3.3853
Hossfeld anamorphic	-	2.7573 (0.0698)	-	-	2.6418 (0.0431)	6.9452
Schumacher polymorphic	4.1458 (0.0226)	-	1.1916 (0.0379)	-	-	3.1562
Schumacher anamorphic	-	8.4717 (0.5834)	0.9740 (0.0515)	-	-	4.5741
Chapman-Richards anamorphic	-	1.5898 (-0.0296)	-0.1158 (0.0068)	-	-	4.7730
Gompertz polymorphic	0.2017 (0.0068)	-	0.0034 (0.0002)	4.0816 (0.0215)	-	3.4263

the range zero when used together with dummy variables. The regions Kiambu, Mt. Kenya and Elburgon were found to differ significantly from the default dummy variable (Uasin Gishu) at 0.05 significant level unlike Nyeri and Njoro whose data were then merged with Uasin Gishu. Coefficients for the final equation (10) are presented below.

$$\hat{G}_2 = G_1^{(T_1/T_2)^\gamma} \exp[(\alpha + \beta_1 k1 + \beta_2 k2 + \beta_3 k3)(1 - (\frac{T_1}{T_2})^\gamma) + \beta_4 Xt(\frac{1}{T_2} - \frac{1}{T_1})\frac{Tt}{T_2} + \beta_5 Xa(\frac{1}{T_2} - \frac{1}{T_1})\frac{T_1}{T_2}] \quad (10)$$

where $\hat{G}_2$ = estimate of basal area at age T_2, G_1 = basal area at age T_1, Tt = thinning age, Xt & Xa = thinning indices, $k1$ = Kiambu, $k2$= Mt. Kenya and $k3$ = Elburgon. The coefficient estimates and their standard error (in brackets) were

$\hat{\gamma} = 1.2344$ (0.0304) $\hat{\alpha} = 4.1996$ (0.0192)

$\hat{\beta}_1 = 0.0727$ (0.0191) $\hat{\beta}_2 = -0.1064$ (0.0309)

$\hat{\beta}_3 = -0.0465$ (0.0227) $\hat{\beta}_4 = 56.0747$ (2.8065)

$\hat{\beta}_5 = 59.3296$ (2.5310) $\hat{\sigma}^2$ (RMS) = 1.8078.

A plot of residual values against predicted values (Figure 5) shows a uniform distribution of residuals without any systematic divergence, an indication of a good fit for the data. The equation predicts basal area ha^{-1} with a mean absolute residual of 0.97 m^2 ha^{-1} which is a 30% improvement compared to 1.38 m^2 ha^{-1} of the Tennent (1990) model.

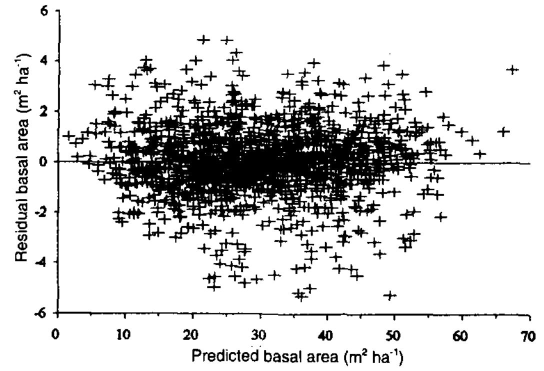

Figure 5. Residual plotted against predicted values (m^2 ha^{-1}) for patula pine basal area ha^{-1} model

Validation of regional predictions

Graphs of predicted and observed mean height and basal area ha^{-1} for cypress and patula pine respectively are shown in Figures 6 and 7. Although the validation data were highly limited, predicted values were very close to those observed for each of the regions. The usefulness of thinning indices was clearly demonstrated by providing adjustments at the point of thinning. Much of the poor fit for the data was associated with outliers resulting from erroneous field measurement, data recording and other handling errors rather than on the model formulations.

Discussion and conclusion

This study has shown that large improvements in growth and yield projections for plantation crops of cypress and pine in Kenya can be realised by modelling each stand variable (mean top height and basal area) separately and using sigmoid projection equations which are biologically consistent and path invariant for different combinations of projection lengths. The use of thinning indices as presented here provides a potential method for improving the fit of basal area models especially in situations where thinning is practised, and data collection after a thinning is not feasible. For models provided here, yield projection from ages T_1 to T_2 and then T_2 to T_3 give the same results as a single projection from ages T_1 to T_3. Review of the Kenya growth models (Tennent 1990) has shown that together with their lack of biological basis and poor fit, a major weakness was that of compounding errors from component equations in every projection (Ngugi 1996).

There were similar growth trajectories for cypress in Uasin Gishu, Kiambu, Nyeri and Njoro while the Elburgon region portrayed a unique trajectory with higher height growth. Mt. Kenya showed unique trends both on height and basal area with lowest yields. All the data used for this region were taken from the dry northwestern side. Data collected from plantations in Meru were combined with those of Nyeri. The patula pine model did not show consistent similarities in growth trends for the two

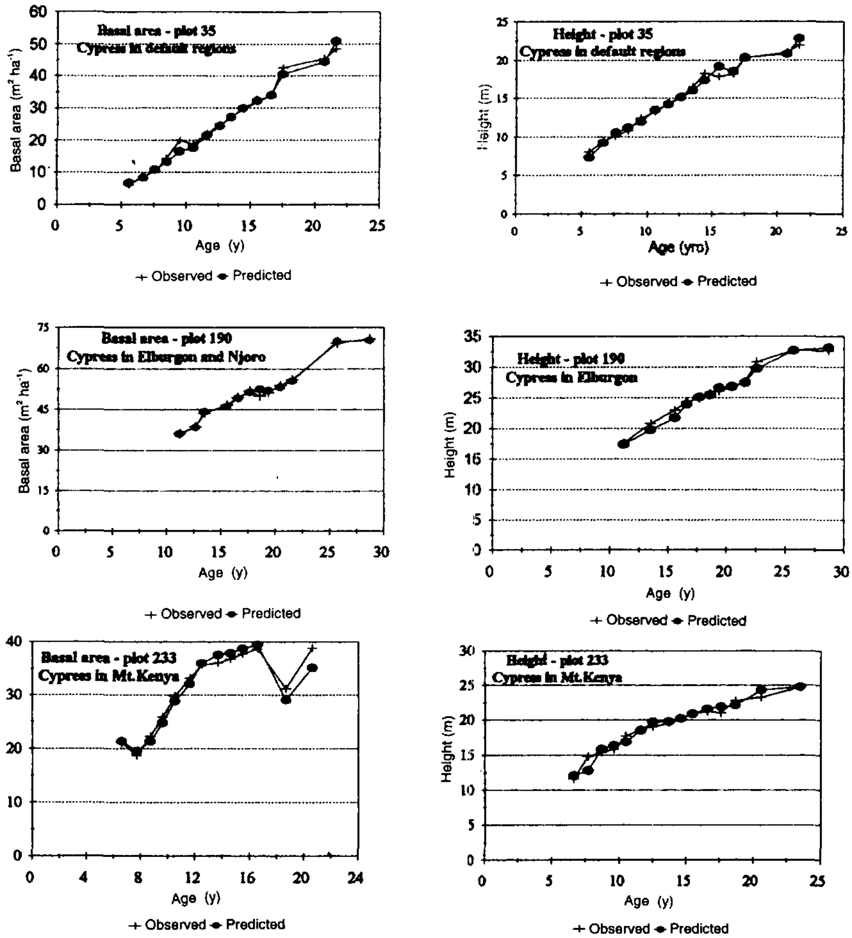

Figure 6. Validation plots for cypress height (m) and basal area (m^2 ha^{-1}) models showing the observed and predicted values against age (y)

stand variables across regions as was found in the cypress model. Basal area trends were found to be quite different across regions. This aspect highlights the danger of using similar coefficients for all regions and especially when sensitive projections are expected.

The normal probability plots of residual errors from height and basal area ha^{-1} equations presented here expressed a slight S-shaped curve as opposed to a straight line for normally distributed residuals. This clearly indicated the presence of outlier data. This problem could be partly associated with the field data collection and processing which having been done over a long period of time involved many personnel some of whom did not have elaborate skills with the measuring instruments. Nevertheless predictions of stand variables in the six forest regions (Uasin Gishu, Elburgon, Njoro, Kiambu, Nyeri and Mt. Kenya) can achieve the following accuracy: cypress mean top height, 0.55 m and basal area, 0.768 m^2 ha^{-1}; and pine species mean top height, 0.64 m and basal area, 0.972 m^2 ha^{-1}.

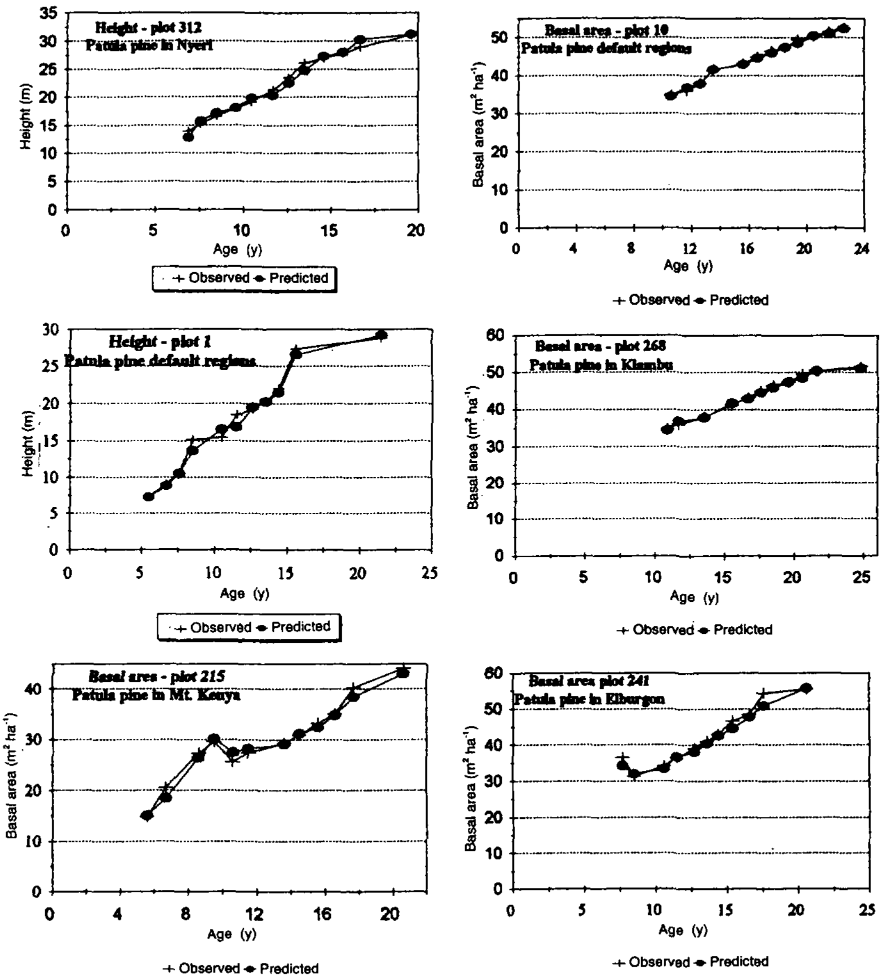

Figure 7. Validation plots for patula pine height (m) and basal area (m^2 ha^{-1}) models showing the observed and predicted values against age (y)

Due to limited data available for Njoro and Mt. Kenya, the results should be applied cautiously. Hence, more data are required for rigorous validation of the models. Data collected from regular planning inventory could tentatively be used awaiting more detailed data from permanent sample plots in the future. The use of dummy variables in this study was mainly to distinguish the geographic locations of the various regions. Other factors affecting tree growth such as biotic, edaphic and climatic factors within each region can be used to refine the models. Altitude height above sea-level was partly used in this study and portrayed some potential to improve the fit of basal area and height equations.

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