GROWTH AND YIELD PREDICTION MODEL OF GAMAR (*GMELINA ARBOREA*) IN CHITTAGONG HILL TRACTS, BANGLADESH

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RAHMAN, M. M. & AHMAD, I. U. 2000. Growth and yield prediction model of gamar (*Gmelina arborea*) in Chittagong Hill Tracts, Bangladesh. Current growth estimation and future yield prediction models of gamar (*Gmelina arborea*) were studied using data from 171 plots in Chittagong Hill Tracts, Bangladesh. All plots were laid at well-stocked gamar plantations of every age class of 1–17 y. Twenty of the 171 sample plots were selected and kept separately to validate the growth and yield models. Data from the remaining 151 plots were used to formulate the models which include stand diameter and height function, number of trees per hectare prediction model, basal area and stand volume equation. The derived models would help to determine the optimal harvest age of gamar plantation and prescribe the best financial investment on forestry business with other competitive uses of land.

Keywords: Growth model - yield prediction - *Gmelina arborea* - Chittagong Hill Tracts - Bangladesh

RAHMAN, M. M. & AHMAD, I. U. 2000. Model ramalan pertumbuhan dan hasil gamar (*Gmelina arborea*) di Chittagong Hill Tracts, Bangladesh. Anggaran pertumbuhan semasa dan model ramalan hasil pada masa hadapan bagi gamar (*Gmelina arborea*) dikaji menggunakan data daripada 171 petak di Chittagong Hill Tracts, Bangladesh. Semua petak ditempatkan di ladang-ladang gamar yang disimpan dengan baik bagi setiap peringkat umur iaitu 1–17 tahun. Dua puluh daripada 171 petak sampel dipilih dan disimpan secara berasingan untuk mengesahkan model pertumbuhan dan hasil. Data daripada 151 petak lagi digunakan untuk merumuskan model yang termasuk garis pusat dirian dan fungsi ketinggian, model ramalan bilangan pokok sehektar, luas pangkal dan persamaan isi padu dirian. Model yang diperoleh dapat membantu untuk menentukan umur tebangan optimum bagi ladang gamar dan menentukan pelaburan kewangan yang terbaik bagi perdagangan perhutanan dengan kegunaan-kegunaan lain yang lebih kompetetif bagi tanah.

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Introduction

Pulpwood plantations have been initiated in many countries to meet the increasing demand of pulp and paper mills while natural sources of raw materials are continuously dwindling. Such a plantation has been initiated since 1974 at Chittagong Hill Tracts, Bangladesh, to supply pulpwood to the Karnaphully Paper Mills. Gamar (*Gmelina arborea* Roxb.) is the main species of the plantation. One study has been made using plantation data to predict growth and yield model of the species for the whole country (Latif *et al.* 1995). But no scientific study has been conducted on the growth prediction and determination of optimal harvest age of gamar particularly for the Chittagong Hill Tracts. The objective of the present study was to develop suitable growth and yield model for the species planted in that region.

'Growth and yield model' is a system of mathematical relationships that provides quantitative descriptions of forest stand development over some range of time, condition and treatment (Curtis & Hyink 1985). The models may be whole-stand, stand-class and single-tree models (Vanclay 1995). Single-tree models simulate the growth of individual trees. In contrast to single-tree models, whole-stand models are fully aggregated approaches to predicting stand growth and yield (e.g. Curtis *et al.* 1981, Demars & Barrett 1987). Stand-class models provide more details by simulating several classes within the stand. The approach is a compromise between single-tree models (a class for each tree) and whole-stand models (a single class for all trees).

Whole-stand models (which have been used in the current study) are conceptually simpler than single-tree models where the stand is the basic modeling unit (Munro 1974). Whole-stand models are generally simpler to understand (Iwasa *et al.* 1987) and easier to develop than more complex models functioning at higher levels of resolution. They are dependent on stand parameters and generally driven by stand density, stand age and site productivity. Stand age can be obtained from plantation records and site quality can be estimated using site-index equations. Stand density may be expressed any number of ways and many of the common measures are closely related (Curtis 1970). Hann and Larsen (1991) used stand basal area as a measure of stand density, whereas Wykoff *et al.* (1982) used crown competition factor (Krajicek *et al.* 1961). Both are based on aggregated transformations of tree diameter (Ritchie & Hann 1997).

All the measurable parameters of a stand are converted to logarithm and reciprocal transformations and used in different combinations with the original forms for predicting growth and yield (e.g. Pimmanrojnagool 1979, Gregorio 1981, Revilla & Gregorio 1982, Gonzales 1985, Latif & Castillo 1990). The estimates of volume and yield of stands with the appropriate economic analysis generate decisions concerning the optimum age to harvest (rotation age) and prescribe the best financial investment on forestry business with other competitive uses of land.

Materials and methods

Study area and plantation

The study area (22° 8'-22° 29' N, 92°10'–92° 21' E) lies in Rangamati and Bandarban districts of Chittagong Hill Tracts. The mean monthly minimum temperature varies from 12 to 25 °C and mean monthly maximum is 25 to 34 °C. The area receives average annual rainfall of 2750 mm with five months having less than 200 mm (Bremar 1986). The area consists of small and medium hills with elevation ranging from 100 to 400 m. The soil of the area is shallow to moderately deep, pH varies from 5.4 to 6.3. The soil is well to excessively drained (Forestal 1966). The ecosystem of the area thrives with tropical wet evergreen and semi-evergreen forests where dipterocarps are the dominant species. Tracts of the degraded forest have been cleared for plantations. Spacing for the plantations was 1.8 × 1.8 m.

Data base

Gamar plantations of ages between 1 and 17 y were selected for measurement on the basis of accessibility and stocking. Data were recorded from 171 plots. The plot size was fixed as 0.01 ha (10 × 10 m) to achieve unbroken micro-topography and uniform stocking. An effort was made to lay the plots in well-stocked portions of the plantations. All the sample plots represent different microtopography of the hills, e.g. position on mountain slope (52 plots hilltop, 82 midslope, 27 hillbase, 10 valley), slope percentage (16 plots flat land, 55 less than 20% slope, 61 between 20 and 40% slope, 39 more than 40% slope) and aspect of the mountain (23 plots open from all aspects, 32 northeast, 34 southeast, 52 southwest, 30 northwest).

Diameter at breast height (dbh) and total height of all trees inside the selected plots were measured by diameter tape and Suunto clinometer respectively. Measured dbh and height were used to calculate tree volume using two-way volume equations (Latif *et al.* 1984). Age was recorded from plantation journals. Site quality of each plot was calculated by following the site-index equation (Rahman & Ahmed 1995) using 15 y reference age:

$$S = H_d\,[\,(1\text{-}e^{-0.055759*15})\ /\ (1\text{-}e^{-0.055759A})\,]^{0.75} \qquad (1)$$

where, S = site index, H_d = dominant height, A = stand age and e = base of natural logarithm.

Precision and acceptability of the yield models can be tested by validation plots. Validation in its purest form requires independent data. The data set may be partitioned with some for development and the rest for validation. A half-and-half split is common in other disciplines (Snee 1977), but fewer data are often used to validate growth models. Goulding (1979) suggested that 10–15 plots spread over a range of stand conditions could suffice if multiple silvicultural options were not being evaluated. The current study used 'leave out more than one observation' approach (Draper & Smith 1981) for growth model formulation and validation. In this approach the idea is to leave out 'm' observations and using 'n-m' remaining observations to construct the model and to validate it by 'm'. Accordingly in the current study, data of 20 plots were randomly selected from the total data set throughout the whole age range and kept separately for validation. Data from the remaining 151 plots were used for computing the growth and yield curves and finally tested by the previously separated 20 plots.

Data processing methodology

Yield prediction models were developed by regression analysis. The models include stand dbh and height function, stand density prediction, basal area and stand volume models. Stand dbh, height and density prediction equations were developed by fitting different equations with all probable combinations of the independent variables. Volume prediction equation was formulated from a methodology developed by Mackinney *et al.* (1937) and Schumacher (1939) and expressed as:

$$\ln(V) = \beta_0 + \beta_1 S + \beta_2 A^{-1} + \beta_3 \ln(B) \qquad (2)$$

where, B = per hectare basal area and V = volume per hectare, α and β = parameters to be estimated.

Basal area model was derived from the equation of Schumacher and Coile (1960) with a slight modification and expressed as:

$$\ln(B) = \alpha_0 + \alpha_1 S + \alpha_2 A^{-1} + \alpha_3 A^{-1} S \qquad (3)$$

Similar type of yield prediction models had also been used for other species, i.e. natural loblolly pine stands in Virginia, South Carolina and Georgia (Sullivan & Clutter 1972), black spruce on organic soil in Minnesota (Perela 1971), etc.

From the equation (3) the following two basal area equations for projected and initial age can be derived:

$$\ln(B_2) = \alpha_0 + \alpha_1 S + \alpha_2 A_2^{-1} + \alpha_3 A_2^{-1} S \qquad (4)$$

$$\ln(B_1) = \alpha_0 + \alpha_1 S + \alpha_2 A_1^{-1} + \alpha_3 A_1^{-1} S \qquad (5)$$

By eliminating α_2 and α_3 the following equation can be derived:

$$\ln(B_2) = (A_1/A_2)\ln(B_1) + \alpha_0(1-A_1/A_2) + \alpha_1 S(1-A_1/A_2) \qquad (6)$$

where, A_1 = initial age, A_2 = projection age, B_1 = initial basal area at age A_1 and B_2 = the predicted basal area at age A_2.

For predicting future volume, equation (2) can be written as:

$$\ln(V_2) = \beta_0 + \beta_1 S + \beta_2 A_2^{-1} + \beta_3 \ln(B_2) \qquad (7)$$

By incorporating equations (6) and (7) with some algebraic rearrangement the following future yield prediction model can be developed:

$$\ln(V_2) = \beta_0 + \beta_1 S + \beta_2 A_2^{-1} + \beta_3 (A_1/A_2)\ln(B_1) + \beta_4 (1-A_1/A_2) + \beta_5 S\,(1-A_1/A_2) \quad (8)$$

All these models were selected on the basis of different precision statistics like *r*-squared, prediction residual sum of squares (PRESS) (Allen & Cady 1982), error sum of squares (Error SS), root mean squared error (RMS Error) and expected shape of the predicted model. The chosen models were checked by independently selected confirmation plots with absolute deviation (%), 45 degree line, *chi*-square test and *t*-test.

Results and discussion

Estimating current yield

The following yield prediction models were developed:

(a) Stand dbh model:

$$\ln(D) = 2.2457 - 2.07237\,A^{-1} + 0.32631\,\ln(S) - 2.3385\,P^{-1} \quad (9)$$

where, P = space per tree.

(b) Stand mean height model:

$$\ln(H) = -\,1.85288 + 0.71541\,\ln(A) + 0.90639\,\ln(S) \quad (10)$$

(c) Number of trees per hectare prediction model:

$$\ln(N) = 8.2951 - 0.133722\,D + 0.035025\,B \quad (11)$$

(d) Stand basal area model:

$$\ln(B) = 2.5780 + 0.0632\,S - 3.7896\,A^{-1} - 0.04051\,S/A \quad (12)$$

(e) Volume yield model:

$$\ln(V) = 1.2176 + 0.022437\,S - 0.6188\,A^{-1} + 1.08438\,\ln(B) \quad (13)$$

where, D = mean stand diameter at breast height in cm, H = mean stand height in m, N = number of trees per hectare, B = per hectare basal area in m^2 and V = volume per hectare in m^3.

The precision statistics and results of validation test of the above equations are given in Tables 1 and 2 respectively. Predicted stand density, mean dbh and height, basal area and volume yield per hectare are represented in Tables 3–5 for different ages and site indices. Figures 1 and 2 show the stand basal area and volume yield models for different site indices respectively.

Table 1. Statistics expressing the precision of growth and yield prediction equations

Prediction model	r-squared (%)	PRESS	Error SS	RMS error	F-value
Dbh	95.20	411.02	4.159	0.1682	979.55[2]
Height	97.30	184.15	2.584	0.1312	2666.67[1]
Density	88.50	363.82×10^5	2.006	0.1164	567.76[1]
Basal area	94.10	637.77×10	15.218	0.3218	778.67[2]
Volume	99.30	647.20×10^2	2.960	0.1419	6661.02[2]

Tabulated F-value is 3.00[1] (numerator *df* 2 and denominator *df* 148) and 2.60[2] (numerator *df* 3 and denominator *df* 147) at 0.05 significant level.

Table 2. Statistics expressing the validation results of the growth and yield equations

Prediction model	Absolute deviation (%)	Slope (degrees)	*chi*-square	*t*-value
Dbh	3.72	45.23	5.84	0.97
Height	4.56	46.63	3.38	1.24
Density	2.80	44.20	5.96	1.18
Basal area	0.71	43.00	0.19	0.09
Volume	1.49	45.10	0.41	0.68

Tabulated value (0.05 significant level, *df* 19) of $t = 2.09$ and *chi*-square = 10.12.

Table 3. Growth and yield of gamar at Chittagong Hill Tracts (site index 9)

Age (y)	Density (n ha^{-1})	Mean dbh (cm)	Mean ht (m)	Basal area (m^2 ha^{-1})	Volume (m^3 ha^{-1})
1	3391	1.3389	1.14874	0.3652	0.7470
2	2943	3.0666	1.88617	2.9145	9.6804
3	2733	4.3811	2.52091	5.8245	22.7376
4	2579	5.4460	3.09699	8.2338	34.8474
5	2442	6.3519	3.63304	10.1347	45.0222
6	2315	7.1479	4.13921	11.6399	53.4069
7	2194	7.8632	4.62181	12.8500	60.3360
8	2082	8.5164	5.08510	13.8395	66.1166
9	1976	9.1201	5.53216	14.6615	70.9929
10	1878	9.6835	5.96527	15.3541	75.1516
11	1786	10.2130	6.38621	15.9450	78.7348
12	1701	10.7137	6.79638	16.4548	81.8510
13	1621	11.1897	7.19692	16.8989	84.5838
14	1546	11.6439	7.58878	17.2890	86.9988
15	1477	12.0791	7.97274	17.6344	89.1475

Table 4. Growth and yield of gamar at Chittagong Hill Tracts (site index 15)

Age (y)	Density (n ha^{-1})	Mean dbh (cm)	Mean ht (m)	Basal area (m^2 ha^{-1})	Volume (m^3 ha^{-1})
1	3262	1.6426	1.8252	0.4184	0.991
2	2763	3.7621	2.9968	3.7711	14.646
3	2569	5.3748	4.0053	7.8479	35.945
4	2436	6.6812	4.9206	11.3213	56.312
5	2315	7.7925	5.7723	14.1053	73.719
6	2196	8.7691	6.5766	16.3320	88.220
7	2080	9.6466	7.3433	18.1346	100.293
8	1969	10.4479	8.0794	19.6160	110.420
9	1862	11.1886	8.7898	20.8513	118.999
10	1761	11.8797	9.4779	21.8954	126.339
11	1665	12.5293	10.1467	22.7884	132.680
12	1576	13.1436	10.7984	23.5603	138.207
13	1493	13.7275	11.7348	24.2338	143.063
14	1414	14.2848	12.0574	24.8265	147.361
15	1341	14.8187	12.6675	25.3518	151.190

Table 5. Growth and yield of gamar at Chittagong Hill Tracts (site index 21)

Age (y)	Density (n ha^{-1})	Mean dbh (cm)	Mean ht (m)	Basal area (m^2 ha^{-1})	Volume (m^3 ha^{-1})
1	3204	1.7930	2.4760	0.4795	1.314
2	2743	4.1066	4.0655	4.8795	22.157
3	2646	5.8669	5.4336	10.5743	56.823
4	2605	7.2929	6.6753	15.5665	90.998
5	2554	8.5060	7.8307	19.6316	120.707
6	2848	9.5719	8.9217	22.9155	145.725
7	2400	10.5298	9.9619	25.5925	166.711
8	2307	11.4045	10.9605	27.8035	184.411
9	2210	12.2130	11.9241	29.6544	199.467
10	2110	12.9674	12.8576	31.2234	212.392
11	2012	13.6765	13.7649	32.5688	223.587
12	1916	14.3471	14.6490	33.7340	233.366
13	1824	14.9844	15.5123	34.7526	241.974
14	1735	15.5927	16.3570	35.6500	249.605
15	1650	16.1754	17.1846	36.4465	256.412

N.B. Mean space per tree of the raw data has been used for mean dbh calculation in the tables.

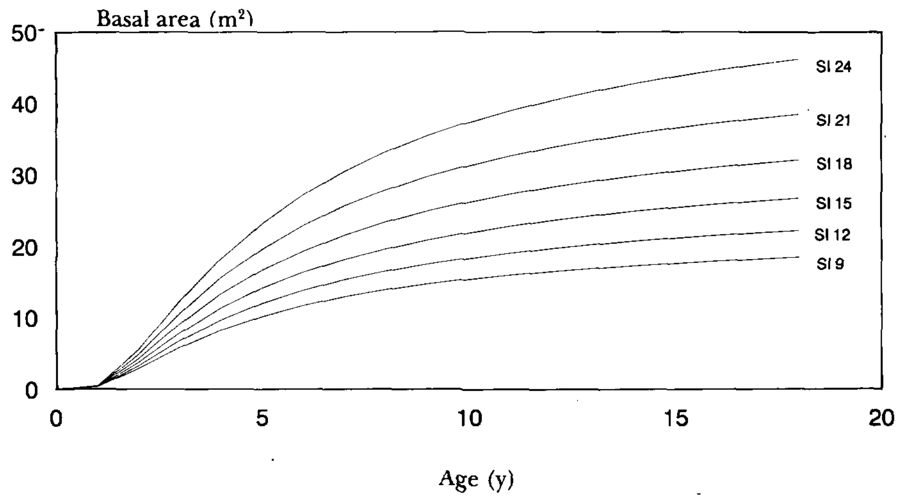

Figure 1. Stand basal area curves of gamar for different site indices (equation 12)

Predicting future yield

Future yield was computed by the projected value of age. The following equations for predicting future basal area and volume were derived from equations 6 and 8 respectively:

$$\ln(B_2) = (A_1/A_2)\ln(B_1) + 2.5780(1\text{-}A_1/A_2) + 0.06320S(1\text{-}A_1/A_2) \qquad (14)$$

$$\ln(V_2) = 1.2176 + 0.022437\ S - 0.6188\ A_2^{-1} + 1.08438(A_1/A_2)\ln(B_1) + 2.7955\ (1\text{-}A_1/A_2) + 0.06853\ S(1\text{-}A_1/A_2) \qquad (15)$$

All the above models can predict the results to less than five percent of deviation. The slopes of regression lines (without having an intercept) of predicted and observed values showed that dbh, height and volume models slightly overestimate, whereas density and basal area models slightly underestimate the results. The computed *chi*-square and *t*-values imply that no significant difference exists between the actual values from the 20 validation sample plots and their corresponding expected values as predicted by the models (Table 2). Derived curves plotted against age provide sigmoid-shaped yield curves. This result supports the biological principle of stand development. Therefore, the derived models can safely estimate the growth and yield of gamar.

These growth models for well-stocked, even-aged, pure gamar stands have the following important applications in forest management: (I) generate decision concerning optimum ages to harvest (rotation ages), (II) determine the best financial strategy from the management options available on any specific parcel of land, and (III) compare the best forestry alternative with other competitive uses of land.

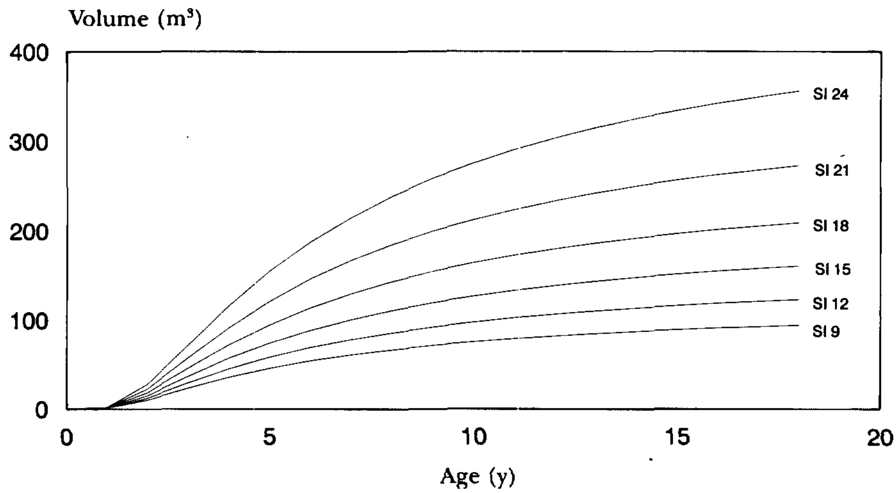

Figure 2. Stand volume yield curves of gamar for different site indices (equation 13)

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