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TAPER AND VOLUME FUNCTIONS FOR *EUCALYPTUS GRANDIS* IN ZIMBABWE

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MABVURIRA, D. & EERIKÄINEN, K. 2002. Taper and volume functions for *Eucalyptus grandis* in Zimbabwe. A polynomial taper curve and a logarithmic volume function for plantation *Eucalyptus grandis* in Zimbabwe were developed from data collected from 589 sample trees at four different sites. The sites covered the whole range of sites where the species is currently grown on a commercial basis. The parameters measured were diameters underbark at stump height and at every 1.5 m along the stem, dbh overbark and underbark, stump height and top section length. Volume predictions by the taper curve and the new volume function were compared with predictions by the old volume function. The polynomial taper curve provided the best volume predictions followed by the new volume function. The new volume function provided better estimates than the old volume function for trees < 30 cm dbh. The old volume function underestimated volumes over all diameter ranges but provided more reliable estimates than the new volume function for trees > 30 cm dbh. The new volume function was recommended for use in pole production while the taper curve was recommended for use in both pole and sawlog production.

Key words: Polynomial taper curve - logarithmic volume function - volume prediction

MABVURIRA, D. & EERIKÄINEN, K. 2002. Keluk tirus dan fungsi isipadu bagi *Eucalyptus grandis* di Zimbabwe. Keluk tirus polinomial dan fungsi isipadu logaritma untuk ladang *Eucalyptus grandis* di Zimbabwe dibangunkan daripada data yang diambil daripada 589 sampel pokok di empat tapak yang berbeza. Tapak tersebut merangkumi semua tapak spesies ini yang ditanam untuk tujuan komersial. Parameter yang diukur ialah diameter bawah kulit pada ketinggian tunggul dan pada setiap 1.5 m di sepanjang batang, diameter aras dada atas kulit dan bawah kulit, ketinggian tunggul dan panjang bahagian atas. Jangkaan isipadu daripada keluk tirus dan fungsi isipadu yang baru dibandingkan dengan jangkaan daripada fungsi isipadu yang lama. Keluk tirus polinomial menyediakan jangkaan terbaik bagi isipadu diikuti dengan fungsi isipadu yang baru. Fungsi isipadu yang baru memberi jangkaan yang lebih baik daripada fungsi isipadu yang lama bagi pokok berdiameter aras dada < 30 cm. Fungsi isipadu yang lama terkurang jangka bagi semua julat diameter tetapi memberikan

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jangkaan yang lebih tepat daripada fungsi isipadu yang baru bagi pokok berdiameter aras dada > 30 cm. Fungsi isipadu yang baru disyorkan untuk pengeluaran tiang sementara keluk tirus disyorkan untuk pengeluaran tiang dan balak gergaji.

Introduction

Eucalyptus grandis is an important species in Zimbabwe, constituting over 90% of the total industrial hardwood plantations area. It is grown for a variety of products such as poles (utility, fencing and construction), mining timber, pulpwood and sawn timber. Hence, accurate determination of its volume is of paramount importance.

Tree volume needs to be accurately determined not only for research, but for management and planning purposes. For species such as *E. grandis* that are managed for a variety of products, an additional requirement is that total tree volume be expressed accurately by product quality and log sortiments. One way of achieving this is to develop flexible taper and volume functions that do not need to be changed when products specifications change (Reed & Green 1984, Heinonen *et al.* 1996, Trincado *et al.* 1996). Volume and taper functions have traditionally been developed from an analysis of sample tree diameters measured at several points along tree stems. The diameters have normally been taken at constant intervals or at prescribed proportional heights along the stem. The form or shape of trees has been approximated to be a cylinder, or combinations of a cylinder, frustum (neoloid and paraboloid) and a cone. Tree volumes were then calculated using formulae such as Huber's, Newton's and Smallian's. The resulting functions mostly predicted volume from a single parameter such as total tree height or diameter at breast height (dbh), or from a combination of both variables. Normally, a form factor was also included in the equations to account for the variability in tree form (Husch *et al.* 1982, Laasasenaho 1982, LeMay *et al.* 1993, van Laar & Akça 1997). Taper functions, expressed as a relationship between height and diameter along the stem, have also been used in the construction of volume tables and in the direct estimation of log and tree volumes.

Over the years, many researchers have modified and improved these basic equation forms in line with advances in statistical and computational methods. Bruce *et al.* (1968), Demaerschalk (1972), Amidon (1984), van Laar & Schonau (1988) applied regression techniques to develop taper and volume functions. Gregoire *et al.* (1986) introduced importance sampling and control-variate sampling methods while Wood *et al.* (1990) advocated the centroid sampling method in the construction of taper functions. Max and Burkhart (1976) and Flewelling and Rynes (1993) used segmented polynomial functions to account for changes in taper along the tree stem while Lahtinen and Laasasenaho (1979) used spline techniques. Kozak (1988) and Newnham (1988, 1992) developed variable-exponent models to describe stem taper while Valentine *et al.* (1993) used the centroid method to estimate the volumes for any log or section of the stem. Robinson and Wood (1994) reviewed the current state-of-the-art techniques of tree volume estimation and recommended systems based on taper functions.

In Zimbabwe, besides the work by Prevôst (1972) on developing volume equations for *Pinus* and *Eucalyptus* species, and by Eerikäinen *et al.* (1999) who compared three types of taper estimation methods for *Eucalyptus cloeziana*, not much has been done. Using over one thousand sample tree data, Prevôst (1972) applied regression techniques to develop a logarithmic volume function (in imperial units) for *E. grandis.* Eerikäinen *et al.* (1999) found the polynomial taper estimation method to be superior to the Schumacher's taper equation and the diameter distribution method.

In view of the importance of *E. grandis* in Zimbabwe, the local timber industry welcomes efforts aimed at evaluating or improving current methods of volume determination for the species (Anonymous 1999). The purpose of this study was to develop volume and taper functions for *E. grandis* grown in Zimbabwe and to evaluate their estimates against current volume equations for the species.

Materials and methods

Data description

Sample tree stem measurements were collected from commercial stands of *E. grandis* at four sites (Mtao, Mountain Home, Imbeza and Ngungunyana) representing a wide range of sites where the species is currently grown (Table 1). The sample trees were measured from plots located randomly in stands that ranged from one year to nine years old, comprising 1371 to 1736 stem ha^{-1}.

Table 1 Description of locations where sample tree data was collected

Location	Altitude (m asl)	Rainfall (mm $year^{-1}$)	Soil description
Mtao	1470	690	Kalahari sands
Mt. Home	1300	1600	Granite derived sandy loams
Imbeza	1200	1600	Granite and dolorite derived clay loams
Ngungunyana	1050	1000	Granite derived clay loams

The sample trees were felled first to enable measurements to be taken. The parameters measured were, diameters under bark at stump height and at every 1.5 m along the stem, dbh overbark (dbh_{ob}) and underbark (dbh_{ub}), stump height and top section length. A total of 589 trees were measured. A random number generator was used to split the total number of sample trees into two data sets (modelling and validation) by diameter class at a ratio of 2:1. The modelling data set was used to develop the models while the validation data set was reserved for validating the models (Figure 1 and Tables 2 and 3). A large proportion of the sample trees was below 25 cm in dbh and 35 m in height.

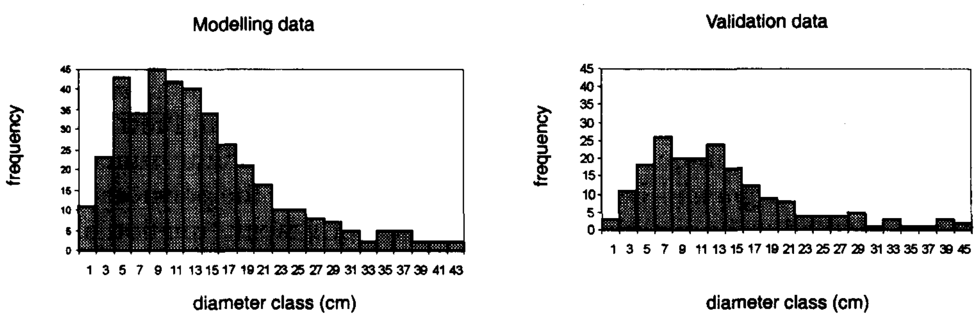

Figure 1 Distribution by diameter class of sample trees of *Eucalyptus grandis*

Table 2 Descriptive statistics of *Eucalyptus grandis* sample tree data

	Modelling data		Validation data	
	dbh_{ob} (cm)	height (m)	dbh_{ob} (cm)	height (m)
N	393	393	196	196
Mean	13.57	15.17	13.67	15.28
SD	8.56	8.99	8.64	8.99
Minimum	1.0	2.1	1.5	2.6
Maximum	43.5	46.0	44.5	47.9

Table 3 Sample trees by diameter and height classes of *Eucalyptus grandis* modelling data

h (m)	dbh_{ob} (cm) 2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	Σ
2	9																						9
4	7	8	2																				17
6	3	11	10	2	1																		27
8		9	11	7	6																		33
10		1	16	12	13	3	4																49
12			1	14	15	15	7	2	2														56
14				4	12	17	8	6	3	1													51
16				1	2	4	13	8	6	4	2												40
18					2	2	0	3	6	5	1												19
20							3	5	2	1	3												14
22						1		6	1	3			1										12
24									2	5	3	5	1	1									17
26										2	1	1	2	3									9
28												1	1	1	2	1							6
30										1	1	1	1	1									5
32															2	1	1		1				5
34												2	1	1	2								6
36																	1		1				2
38														1			1						2
40																		1					1
42															1	1			2			1	5
44																		2		1		1	4
46																	1		2	1			4
48																							
50																							
Σ	19	29	40	40	51	42	35	30	22	22	11	10	7	8	7	4	3	3	6	2		2	393

h = height

Volume estimation

The cubic spline method, recommended by Lahtinen and Laasasenaho (1979), Pukkala and Pohjonen (1990) and also applied by Heinonen *et al.* (1996) and Eerikäinen *et al.* (1999), was used to compute the total stem volumes of sample trees. According to this method, three auxiliary points, additional to stem measurements obtained in the field, are required for the spline curve to pass through all measured points along the stem. The first auxiliary point was set below ground level at an imaginary point of a distance equal to stump height. The exact point was determined by interpolating the path of a parabola passing through the stump, dbh (1.3 m) and at 3.0 m height. The second auxiliary point was at the tip of the tree (diameter equals zero). The third auxiliary point was imagined to be above the tree tip at a point that has the same but negative diameter as the last measured diameter. A spline curve was then fitted to the measurements, including the auxiliary points, for each sample tree (Figure 2).

The stump height of trees had a positive correlation with dbh. Therefore, dbh was used as a predictor in the linear model of stump height. All volume estimations, therefore, began from the estimated stump height. The volume underbark of each sample tree was accurately determined by firstly envisaging the stem to be a series of cylinders 1-cm high and secondly, calculating the volume of each of these cylinders above the stump, using numerical integration, and summing them up. Diameters for the cylinders were read from the spline curve (Press *et al.* 1992). For further applications of the polynomial taper curve, a model to convert overbark dbh to underbark was also developed.

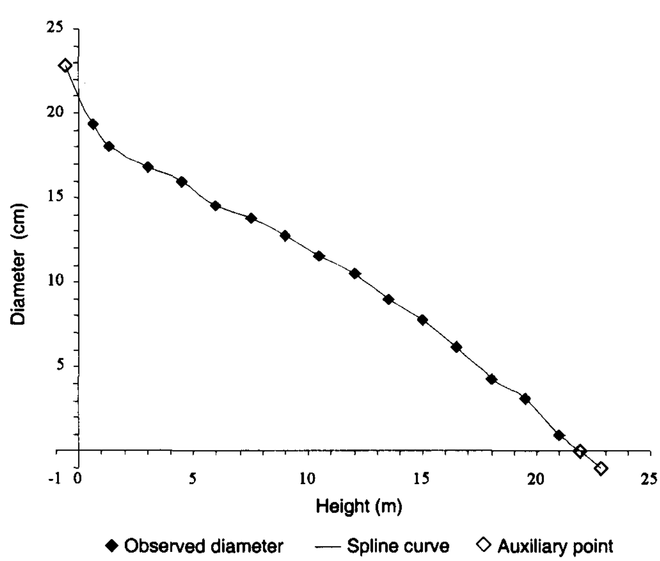

Figure 2 Spline interpolation of tree taper using observed diameters and auxiliary points

The generalised-least-squares (GLS) estimator was applied in the estimation of regression model parameters, due to the hierarchical nature of the data (Lappi 1986). The PROC-MIXED procedure of SAS software (SAS/STAT 1992, Release 6.07) was used to estimate both fixed and random parameters of these mixed models. Accordingly, the residual variation was divided into two components: between (b_{ij}) and within stand (e_{ij}) effects. The site effect was also tested as one of the hierarchical levels, but was found to be statistically non-significant. All calculations were done using the fixed part of the models, with appropriate correction factors for logarithmic-based models.

After several trials with volume functions that use dbh and height as predictors, the logarithmic model form was selected ahead of other alternative model forms. Several transformations of the predictors including weighting were also applied. The volume models were evaluated against the logarithmic volume equation (Prevôst 1972). The equation is

$$\lg_{10}(V) = \left\{ \begin{array}{l} -2.821875861 + 1.764419111 \cdot \lg_{10}\left(\dfrac{\mathrm{dbh}_{ob}}{2.54}\right) + \\ 1.171024768 \cdot \lg_{10}\left(\dfrac{h}{0.3048}\right) + 0.5 \cdot (0.016342)^2 \cdot \ln(10) \end{array} \right\} . 28.3168 \qquad (1)$$

where

$\lg_{10}(V)$ = logarithm to base 10 of volume in dm^3,
dbh_{ob} and h = diameter overbark at breast height in cm and height in m respectively.

Taper curve estimation

A polynomial model with powers that conform to the Fibonacci series was used to develop the taper curve as recommended by Laasasenaho (1982, 1993). The model is of the form:

$$d_l / d_{0.2h} = c_1x^1 + c_2x^2 + c_3x^3 + c_4x^5 + c_5x^8 + c_6x^{13} + c_7x^{21} + c_8x^{34} \qquad (2)$$

where

d_l = diameter at a height l from the ground
$d_{0.2h}$ = basic diameter at 20% of total height h and
x = relative distance from the top or $(1 - (l/h))$.

The basic relative taper curve was then calculated using weighted means of relative diameters at given relative heights using the formula provided by Laasasenaho (1982):

$$t_l = \frac{\left(\frac{1}{n}\right)\sum_{j=1}^{n} d_{lj}}{\left(\frac{1}{n}\right)\sum_{j=1}^{n} d_{0.2h}} = \frac{\sum_{j=1}^{n} d_{0.2h} \cdot \frac{d_l}{d_{0.2h}}}{\sum_{j=1}^{n} d_{0.2h}} = \frac{\bar{d}_l}{\bar{d}_{0.2h}} \tag{3}$$

where

n = number of trees and
t_l = basic or proxy taper curve.

The basic taper curve was then adjusted to account for probable differences in relative heights of trees of the same dbh but different height by correction polynomials that were constrained to pass through three additional points (additional to the points at 20 and 100% (top) of height). The aim was to account for the differences between the basic curve and the observed curve. Thus, the difference between the basic and observed curves at 20 and 100% of height had to be zero. The best correction polynomial was found to be of the fourth order passing through 2.5, 20, 30, 70 and 100% of height. The coefficients of the correction polynomial were added to the coefficients of the basic taper curve to give the adjusted taper curve.

In equations (2) and (3), let $d_l/d_{0.2h} = f(x) = f(1-(l/h)) = t_l$. The volume of the interval $l_1 - l_2$ is obtained from the integral:

$$v = \frac{\pi}{4} \cdot d^2_{0.2h} \cdot h \cdot \int_{1-\frac{l_2}{h}}^{1-\frac{l_1}{h}} \left(f(x)\right)^2 dx \tag{4}$$

The total tree volume to tip is therefore obtained by setting the upper limit of the integral equal to total tree height (Laasasenaho 1993). The taper curve predictions were compared with the predictions by the new (ours) and old (Prevôst's) volume functions as stated above.

Validation and reliability tests

Both volume and taper functions were validated using the validation data set initially reserved for the purpose. Results of the basic and validation data sets were compared. Model bias was evaluated by calculating the means and root mean square errors (RMSE) of the differences between observed and estimated volumes using the following formulae:

$$\mathrm{BIAS} = \left(\frac{1}{n}\right) \cdot \sum_{i=1}^{n} (y_i - \hat{y}_i) \tag{5}$$

$$\mathrm{BIAS}_{\%} = 100 \cdot \left(\left(\frac{1}{n}\right) \cdot \sum_{i=1}^{n} (y_i - \hat{y}_i) \right) \Big/ \left(\left(\frac{1}{n}\right) \cdot \sum_{i=1}^{n} \hat{y}_i \right) \tag{6}$$

$$\mathrm{RMSE} = \sqrt{\frac{n \cdot \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 - \left(\sum_{i=1}^{n} (y_i - \hat{y}_i) \right)^2}{n \cdot (n-1)}} \tag{7}$$

$$\mathrm{RMSE}_{\%} = 100 . \sqrt{\frac{n \cdot \sum_{i=1}^{n} (y_i - \hat{y}_i)^2 - \left(\sum_{i=1}^{n} (y_i - \hat{y}_i) \right)^2}{n \cdot (n-1)}} \Big/ \left(\left(\frac{1}{n}\right) \cdot \sum_{i=1}^{n} \hat{y}_i \right) \tag{8}$$

where

y and $\hat{y}$ = observed and estimated volumes respectively,
BIAS and RMSE = average means and standard deviations of volume differences respectively and
$\mathrm{BIAS}_{\%}$ and $\mathrm{RMSE}_{\%}$ = BIAS and RMSE expressed as percentages.

Results

Models

The stump height model was:

$$h_{\mathrm{St}} = 9.21249 + 1.47700 \cdot \ln(\mathrm{dbh}_{\mathrm{ob}}) \tag{9}$$

where

h_{St} = stump height (m) and
$\mathrm{dbh}_{\mathrm{ob}}$ = diameter overbark at breast height (1.3 m).

The final volume function was based on the natural logarithms of height and diameter and weighted by height (equation (10)) as follows:

$$\ln(v_{\mathrm{ub}}) = c_0 + c_1 \cdot \ln(\mathrm{dbh}_{\mathrm{ob}}) + c_2 \cdot \ln(h) + c_3 \cdot \ln\left(\frac{\mathrm{dbh}_{\mathrm{ob}}}{h - 1.3}\right) + b_i + e_{ij} \tag{10}$$

where

v_{ub} = volume underbark (dm^3),
dbh_{ob} = overbark diameter (cm) at breast height (1.3 m),
h = height (m),
b_i = random stand effect,
e_{ij} = random error term of tree j on stand i and
$c_0 \ldots c_3$ = parameters to be estimated.

For the above logarithmic model (equation (10) and Table 4), the correction factor $((\delta_s^2 + \delta_e^2)/2)$ is 0.00677. An underbark volume table derived from the above volume function is presented in Table 10.

Table 4 Statistics for the volume equation (10)

Parameter	Estimate	SE
c_0	- 3.86525	0.07021
c_1	0.38852	0.09551
c_2	2.68085	0.11562
c_3	1.34952	0.09552
δ_s^2	0.00148	0.00064
δ_e^2	0.01206	0.00091

δ_s^2 is random stand effect, δ_e^2 is random error term, SE is standard error of parameter estimate

The dbh underbark model was

$$\mathrm{dbh}_{ub} = c_1 \cdot \mathrm{dbh}_{ob} + b_i + e_{ij} \tag{11}$$

where

dbh_{ub} = diameter underbark at breast height (cm).

The standard errors of parameter estimates for the underbark model and their standard errors are presented in Table 5.

Statistics for parameter estimates of the basic polynomial taper curve (equation (2)) are presented in Table 6.

Table 5 Statistics for the dbh underbark (equation (11))

Parameter	Estimate	SE
c_1	0.90362	0.00229
δ_s^2	0.02771	0.01200
δ_e^2	0.25798	0.01934

Table 6 Ordinary least square parameter estimates for the taper polynomial (equation (2))

Parameter	Coefficient	SE
c_1	1.91292	-
c_2	- 0.70560	0.08765
c_3	- 0.69575	0.19383
c_4	1.30313	0.27587
c_5	- 1.11789	0.30036
c_6	0.73358	0.23227
c_7	- 0.26065	0.12675
c_8	0.18482	0.03764

The associated correction polynomials at 2.5, 30 and 70% of total height are indicated in Table 7.

Table 7 Statistics for the correction polynomials of the basic taper function

Model variable	$f(x)_{0.025}$ Coefficient	SE	$f(x)_{0.30}$ Coefficient	SE	$f(x)_{0.70}$ Coefficient	SE
Intercept	- 0.05630	0.02642	- 0.15636	0.03421	- 0.48272	0.10513
$1/dbh_{ub}$	0.41114	0.03688	-	-	-	-
$1/h$	-	-	1.23050	0.12640	-	-
dbh_{ub}^2	-	-	- 0.00005	0.00002	-	-
$dbh_{ub}/(h-1.3)$	0.04200	0.02518	- 0.37728	0.05506	0.27104	0.10712
$dbh_{ub}^2/(h-1.3)^2$	-	-	0.09185	0.02371	- 0.09170	0.03676
$\ln(dbh_{ub})$	-	-	0.13972	0.01618	- 0.25138	0.04654
$\ln(h)$	-	-	-	-	0.34541	0.05735
δ_s^2	0.00188	0.00080	0.00019	0.00008	0.00067	0.00022
δ_e^2	0.00917	0.00071	0.00131	0.00010	0.00258	0.00020

dbh_{ub} is underbark diameter at breast height, cm; h is total tree height, m

Although correction polynomials occasionally give fairly large values for diameter differences, this was not observed in our data. Hence, minimum and maximum values of diameter differences were set at - 0.1 and 0.1 respectively for correcting the basic taper polynomial and estimated by the equations in Table 7 (Laasasenaho 1982).

Reliability test

Results for the reliability tests are summarised in Tables 8 and 9, and in Figure 3. Overall, the taper function provided the best volume predictions, followed by the new and old volume functions respectively. In terms of relative root mean square error only, the taper function still provided the best volume predictions, followed

Table 8 Overall absolute (dm^3) and relative (%) root mean square errors (RMSE), mean biases (BIAS) and rankings in reliability for volume predictions by the taper (Taper) function, new (VF_{new}) and old (VF_{old}) functions. N_b and N_v are total number of sample trees for the modelling and validation data sets respectively

			Modelling material			Validation material				Modelling material			Validation material			
N_b	N_v		RMSE	$RMSE_{\%}$	Rank	RMSE	$RMSE_{\%}$	Rank	Overall rank	BIAS	$BIAS_{\%}$	Rank	BIAS	$BIAS_{\%}$	Rank	Overall rank
393	196	Taper	30.09	15.88	1	27.19	14.04	1	1	2.05	1.08	2	1.32	0.68	1	1
		VF_{new}	32.07	16.63	2	39.96	20.21	3	2	- 1.38	- 0.71	1	- 5.36	- 2.71	2	1
		VF_{old}	39.65	22.03	3	28.94	15.70	2	2	11.56	6.42	3	8.09	4.39	3	3

Table 9 Absolute (dm^3) and relative (%) root mean square errors (RMSE), biases (BIAS) and rankings in reliability for volume predictions by diameter ranges using the taper (Taper) function, new (VF_{new}) and old (VF_{old}) functions. N_b and N_v refer to the total number of sample trees for the modelling and validation data sets, respectively

				Modelling material			Validation material				Modelling material			Validation material			
dbh-class	N_b	N_v		RMSE	$RMSE_{\%}$	Rank	RMSE	$RMSE_{\%}$	Rank	Overall rank	BIAS	BIAS%	Rank	BIAS	$BIAS_{\%}$	Rank	Overall rank
0.0–9.9	156	78	Taper	1.5	10.71	3	2.02	14.47	2	2	- 0.12	- 0.85	1	- 0	- 0.25	1	1
			VF_{new}	1.49	10.38	1	2.03	14.28	1	1	- 0.23	- 1.58	2	- 0.3	- 2.35	2	2
			VF_{old}	1.44	10.65	2	2.01	14.96	3	2	0.6	4.41	3	0.49	3.67	3	3
10.0–19.9	163	78	Taper	9.86	10.01	2	9.07	9.54	2	2	- 0.83	- 0.85	1	- 0.9	- 0.96	1	1
			VF_{new}	9.73	9.85	1	8.88	9.29	1	1	- 1.09	- 1.11	2	- 1.3	- 1.35	2	2
			VF_{old}	9.98	10.52	3	9.21	10.03	3	3	2.78	2.93	3	2.39	2.6	3	3
20.0–29.9	51	25	Taper	39.03	8.88	2	35.31	7.12	1	1	9.32	2.12	2	17	3.42	2	2
			VF_{new}	38.78	8.81	1	37.86	7.58	2	1	8.58	1.95	1	13.1	2.62	1	1
			VF_{old}	44.75	10.77	3	42.98	9.2	3	3	35.59	8.09	3	45.7	9.77	3	3
30.0–39.9	23	11	Taper	106.84	7.27	1	81.91	5.4	1	2	19.45	1.32	1	- 55	- 3.62	2	1
			VF_{new}	113.57	7.46	2	114.2	7.25	3	1	- 33.25	- 2.18	2	- 113	- 7.19	3	3
			VF_{old}	110.15	7.92	3	82.46	5.71	2	3	99.22	7.14	3	82.5	1.32	1	2

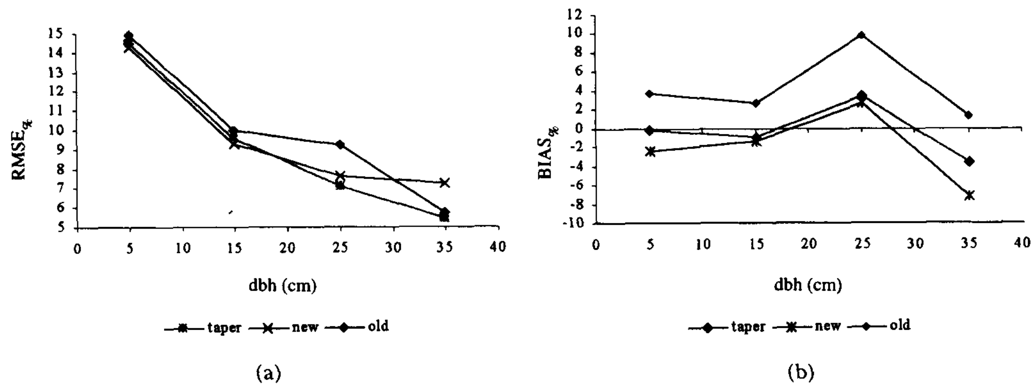

Figure 3 Comparison by diameter classes for the taper and volume functions based on (a) $RMSE_{\%}$ and (b) $BIAS_{\%}$. Taper, new and old are taper function, new and old volume functions respectively

by the old and new volume functions respectively. However, the new volume function was the least biased followed by the taper function and the old volume function respectively (Table 8). When volume prediction comparisons were made by diameter classes, the new volume function provided the best estimates, in terms of relative root mean square error, for diameters below 30 cm dbh while the taper function provided the best estimates for diameters above 30 cm dbh (Table 9). The new volume function ranked second, together with the old volume function, for estimates above 30 cm dbh. The taper function was also the least biased for diameters below 20 cm, but ranked second (ahead of the new volume function) for diameters above 30 cm. While the new volume function provided the best estimates for trees of dbh 30 cm and below, the difference with taper function estimates was relatively small. This is clearly demonstrated in Figure 3(a). Figure 3(b) also demonstrates that the old volume function consistently provided underestimates for the whole range of diameter classes of the data set while the taper and new volume functions overestimated the volume of small trees (less than 20 cm dbh) and large trees above 30 cm dbh.

Discussion and conclusions

This study showed that the polynomial model form can reliably be applied to *E. grandis* plantations to describe tree taper. After Laasasenaho (1982) successfully applied it on several temperate hardwood and softwood species and Heinonen *et al.* (1996) adopted it for several exotic plantation species in Zambia, the results of this study are a further testimony to the flexibility of the polynomial model form in describing tree taper for a wide range of species and environments. Recently, Eerikäinen *et al.*(1999) also confirmed the suitability of this model form to plantation-grown *E. cloeziana.*

Table 10 Underbark volume table in decimetres for *Eucalyptus grandis* in Zimbabwe

Dbh (cm)	Height (m)																								
	2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
2	1	1	1	1	2																				
4	2	3	4	5	6	7	9																		
6	5	5	7	10	12	15	18	21	25																
8	8	8	12	16	20	25	30	35	41	46	52	58													
10	12	12	17	23	30	37	44	52	60	68	77	86	95	104	113										
12	16	17	24	32	41	51	61	71	82	94	105	118	130	143	156	169	183	197							
14		22	31	42	54	66	79	93	108	122	138	154	170	187	204	221	239	257	275	294	313				
16			39	53	68	83	100	117	136	154	174	194	214	235	257	279	301	324	347	371	395	419	444		
18					83	102	123	144	166	189	213	238	263	289	315	342	370	398	426	455	485	515	545	576	607
20							147	173	200	228	256	286	316	347	378	411	444	478	512	547	582	618	655	692	729
22							174	204	236	269	302	337	373	409	447	485	524	564	604	645	687	729	773	816	861
24								238	274	312	352	392	434	476	520	564	609	656	703	751	799	849	899	950	1001
26									315	359	404	451	498	547	597	648	700	753	808	863	918	975	1033	1091	1150
28										408	460	512	567	622	679	737	797	857	919	981	1045	1109	1175	1241	1309
30											518	578	639	702	766	831	898	966	1036	1106	1178	1251	1324	1399	1475
32											580	646	715	785	857	930	1005	1081	1159	1237	1318	1399	1482	1566	1650
34												718	794	872	952	1033	1116	1201	1287	1375	1464	1555	1646	1740	1834
36												793	877	963	1051	1141	1233	1326	1422	1519	1617	1717	1818	1921	2025
38													964	1058	1155	1254	1354	1457	1562	1668	1776	1886	1997	2110	2225
40													1053	1157	1262	1370	1481	1593	1707	1824	1942	2062	2184	2307	2432

Stem volumes (dm^3) are predicted using the new volume function (equation (10)).

As expected, the taper curve provided the best volume estimates compared with the new and old volume functions. However, its mean bias and standard error could have even been smaller had more stem measurements been taken below 1.3 m and more sample trees of large diameters measured. The lower section of the stem is normally difficult to model. To minimise errors in volume estimation, sample measurements in this lower section are recommended to be taken at small intervals (Trincado *et al.* 1996). In our data set, sample measurements in this lower section (between stump and breast height) were inadequate as stem measurements were taken at regular intervals of 1.5 m.

The basic logarithmic form using dbh and height as the predictor variables provided good estimates of volume. Overall, the new (weighted) volume function was better than the old (unweighted) volume function. However, because the sample size of large diameter trees was too small, as indicated in Figure 1 and Table 3, the reliability of volume estimates for large diameter trees by the new volume functions was compromised. In practice, however, this will not have much effect on volume estimates as *E. grandis*, mostly grown for poles in Zimbabwe, is not normally left to attain sizes greater than 30 cm dbh before harvesting. Nonetheless, there is scope for further improvements to the new volume function to enable it to reliably cater for large diameter trees, in case management emphasis shifts from the current pole regime to sawlog regime. The old volume function is therefore recommended for use in estimating volumes for sawlog regimes (dbh greater than 30 cm) while the new volume function is recommended for use in pole regimes (dbh less than 30 cm). It is hoped that timber growers in Zimbabwe will take full advantage of these new developments in determining tree volume for *E. grandis*.

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