GROWTH AND YIELD FUNCTIONS FOR *DALBERGIA SISSOO* PLANTATIONS IN THE HOT DESERT OF INDIA GROWN UNDER IRRIGATED CONDITIONS

V. P. Tewari* & V. S. K. Kumar

Arid Forest Research Institute, P.O. Krishi Mandi, New Pali Road, Jodhpur – 342 005, India

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TEWARI, V. P. & KUMAR, V. S. K. 2005. Growth and yield functions for *Dalbergia sissoo* plantations in the hot desert of India grown under irrigated conditions. *Dalbergia sissoo* forms a major portion of irrigated plantations in the hot desert of India. For effective management of these plantations growth and yield functions are essential. This paper describes and reports the development of top height, diameter and volume growth functions based on analysis of data collected from permanent sample plots laid out at the study area. Some base-age variant and invariant site-index models were applied to the data set and compared for relative accuracy. Base-age invariant models produced better results. Based on fit statistics, the Goelz–Burk model was selected for developing a top height model/site index equation to generate site information. The Chapman–Richards, Gompertz and Schumacher models were used for developing diameter–age and volume yield–age functions for stands of *D. sissoo* of known stand density and site class. Among the three models tried, the Gompertz function was found most suitable for the prediction of diameter and volume growth in *D. sissoo* plantations at the study area.

Key words: Base-age – variant – invariant – site index equations – diameter – age equations – volume – age functions – *Dalbergia sissoo* – hot arid region – Rajasthan

TEWARI, V. P. & KUMAR, V. S. K. 2005. Fungsi pertumbuhan dan fungsi hasil untuk ladang *Dalbergia sissoo* yang dilengkapi sistem pengairan di padang pasir yang panas terik di India. *Dalbergia sissoo* merupakan tanaman ladang utama di padang pasir yang panas terik di India. Fungsi pertumbuhan dan fungsi hasil adalah penting bagi memastikan pengurusan ladang yang berkesan. Kertas kerja ini menerang dan melaporkan tentang perkembangan fungsi pertumbuhan ketinggian dominan, diameter dan isi padu berdasarkan analisis data yang dikumpul daripada plot sampel tetap di kawasan kajian. Beberapa model indeks tapak yang berubah dan tak berubah terhadap pangkal-usia diaplikasikan kepada set data dan dibandingkan untuk menentukan ketepatan relatifnya. Model yang tak berubah terhadap pangkal-usia menghasilkan keputusan yang lebih baik. Berdasarkan statistik padanan, model Goelz dan Burk dipilih untuk membangunkan persamaan model ketinggian dominan/indeks tapak bagi menjana maklumat tentang tapak. Model Chapman-Richards, Gompertz dan Schumacher diguna untuk membangunkan fungsi diameter-usia dan fungsi hasil isi padu-usia untuk dirian *D. sissoo* yang diketahui ketumpatan serta kelas tapaknya. Antara ketiga-tiga model ini, fungsi Gompertz didapati paling sesuai untuk meramal diameter serta pertumbuhan isi padu ladang *D. sissoo* di kawasan kajian.

*E-mail: vptewari@afri.res.in

Introduction

To combat desertification in Rajasthan State of India, the State Forest Department has taken up massive afforestation activities along the Indira Gandhi main canal and its distributaries by planting various tree species including *Dalbergia sissoo*. *Dalbergia sissoo* fetches a high price in the domestic market where it is extensively used as timber for making furniture.

In view of the remarkable progress in the field of site productivity evaluation, it is pertinent that adequate work is yet to be taken up for tree species in India (Ray 1994). Site quality prediction systems are generally integrated with growth and yield estimation systems. No information is available for the many plantation species, which are gradually assuming importance. As a result, it is difficult to select sites suitable for a particular species or alternative ones. In the absence of sufficient information, it is difficult to objectively decide on an appropriate rotation length. The importance of site information increases exponentially, especially if cultural practices are applied at the time of stand establishment when response is dependent on site factors.

The Forest Department needs scientific information for the proper management of its productive resource. The information available on the growth and yield aspects of *D. sissoo* is meager. Hence the development of growth and yield functions for *D. sissoo* based on permanent sample plots is deemed very important.

Establishment

The most common form of establishment of *D. sissoo* in the study area is through raising the seedlings in polybags and root trainers in the nursery and planting in the field into pits at the onset of the monsoon. During 1970–80, the planting density was approximately 2500 to 3000 stems per hectare at about 2 x 2 m spacing. Later the spacings were changed to 2 x 3 and 3 x 3 m. During the last 4–5 years, the spacings have been raised to even at 3 x 5 and 5 x 5 m. The rotation length of this species in India is usually 25–40 years depending upon site, climate and the type of plantation (Tewari 1994).

Site quality and growth functions

An effective estimation of site quality is very important for forest management. Site index has been the most widely used means for estimating potential forest site productivity (Payandeh & Wang 1994). Site index may be defined as the height of the trees that are dominant or co-dominant and healthy at a predetermined age referred to as the base or index age, which is somewhat less than the rotation age (Goelz & Burk 1992). The top height of a given species at a certain age is more closely related to the productive capacity of a given site to produce wood of that species than any other single measure (Spurr & Barnes 1980). Height growth is generally the most stable directly measured stand growth statistic. Top height growth is independent of stocking over a fairly wide range of stand density and thus is frequently used as a measure of site productivity (Monserud 1984).

The classification and use of growth models for the management of forest stands have been well documented over the years by several workers (viz. Munro 1974, Mohren & Burkhart 1994, Philip 1994, Gadow & Hui 1999). Mohren and Burkhart (1994) in their work have emphasized the relevance of the management models to the forest managers who require robust models that give reliable predictions within specified limits. Such models become more complex as the range of management options widens. Ease of operation is an important consideration in the development of growth and yield models. To be more useful, a model should be based on easily and cheaply determined stand parameters (Phillips 1995).

The objective of the present study was to develop top height model/site index equation and growth and yield functions for *D. sissoo* that can be used by the forest managers for the management of plantations of this species in the study area.

Materials and methods

Study area

The mean monthly maximum temperature in the area varies between 39.5 and 42.5 °C while the mean monthly minimum temperature varies between 14 and 16 °C. The mean annual rainfall in the area varies from 120 to 300 mm. The major quantity of rainfall is received during the southwest monsoon season (July–September). The number of rainy days varies from 8 to 17 days in the area. The mean monthly relative humidity in the Indira Gandhi Nahar Project (IGNP) area fluctuates largely during the year from 15 to 80%. The mean evaporation varies from 2.7 to 4.7 mm per day in winter and from 13.2 to 15.3 mm per day in summer with the annual mean being of the order of 7.3 to 8.5 mm per day. Wind speeds as high as 130 km per hour have been experienced during the summer months. Dust storms are also common in the region (3–17 days per year). The terrain of the area is very undulating consisting of moving sand dunes, dry undulating plains of hard sand and gravelly soil and rolling plains of loose sand. The soil is rich in potash but poor in nitrogen and low in organic matter with very low productivity. There is presence of semi-consolidated lime concretionary or gypsum strata in many places. The soils are coarsely textured with a low water retention capacity.

Data and field procedure

Permanent sample plots (PSP) were laid out at various locations throughout the IGNP area, covering the available age groups, stand densities and sites, using stratified multistage sampling. Each plot was representative of the growing conditions in the stand, and was of size 0.1 ha. A total of 30 sample plots were laid out for this study. For identification and demarcation, trenches were dug at the four corners of the plot and the trees inside the plot were numbered and enumerated.

The check trees surrounding the plots were marked with rings. The study was started in 1995 and measurements were taken in the sample plots annually for five years. Out of the 30 plots, 8 plots (plots No. 1–8) were laid out in the first year, 13 plots (plots No. 9–21) in the second year and 9 plots (plots No. 22–30) in the third year of the experiment and these plots had 5, 4 and 3 annual measurements respectively. The PSP data included records of age, dominant height, average height, number of trees per hectare, top diameter, basal area per hectare, quadratic mean diameter and timber and wood volume (over bark and under bark) per hectare.

Two-phase sampling was adopted for enumeration of trees in each plot. First the diameter at breast height (dbh) of all the trees within the plots was measured and then a small subsample of diameter–height pairs was taken which was subsequently used to determine the height (*H*) and dbh (*D*) regression. The height of the other trees in the sample plots was estimated using this regression equation. Trees, representing different diameter classes in the plots, were felled from the area surrounding the plots and measured for *D*, *H* and timber and wood volumes (over bark and under bark). Equations showing the relationship between volumes and D^2H were derived for volume prediction, which was applied on the trees within the plots to estimate wood volume per hectare. The PSP data are summarized in Table 1. The growth functions were developed for dominant height, diameter and volume yield using SPSS statistical software package.

Dominant height

Over the last few decades, biological growth functions have been used to describe height growth patterns. Several models based on some extension of the Chapman–Richards function are reported in the literature. The following three models have been used to develop site index equations and based on the fit statistics the best model was selected:

Ek (1971):

$$H = aS^{b}\left(1 - e^{-ct}\right)^{dS^{e}} + \varepsilon \tag{1}$$

Goelz & Burk (1992):

$$H_2 = H_1\left[\frac{1 - \exp\left\{-a\left(\frac{H_1}{t_1}\right)^{b} t_1^{c} t_2\right\}}{1 - \exp\left\{-a\left(\frac{H_1}{t_1}\right)^{b} t_1^{c} t_1\right\}}\right]^{d} + \varepsilon \tag{2}$$

Payandeh & Wang (1994):

$$H_2 = aH_1\left(1 - e^{-ct_2}\right) + \varepsilon \tag{3}$$

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Table 1 Summary of the *Dalbergia sissoo* permanent sample plots in Indira Gandhi Nahar Project (IGNP) area

Plot No.	Site index (m)**	Age (years)		Stocking (stems ha^{-1})		Average height (m)		Dominant height (m)		Average diameter (cm)		Volume per ha (m^3)	
		Initial	Final	Initial	Final	Initial	Final	Initial	Final	Initial	Final	Initial	Final
Site class I (16.7 to 18.7 m)													
6*	17.81	10.2	11.5	2537	2502	9.71	10.83	14.54	15.62	9.98	10.46	93.65	111.96
9	17.17	8.3	11.3	1902	1891	9.89	11.45	12.34	14.96	9.94	11.68	62.72	111.14
10	18.68	8.3	11.3	1644	657	13.98	15.78	18.21	18.48	13.51	16.52	165.20	109.61
16	18.32	9.5	12.6	1356	1149	11.09	13.15	14.72	17.10	12.94	14.72	99.90	125.63
18	17.70	8.5	11.6	1438	1292	9.89	11.75	14.25	16.49	12.25	14.20	84.00	120.22
19	17.64	8.5	11.6	1599	1429	12.10	13.19	16.83	17.30	13.33	14.63	136.33	154.77
21	17.19	6.5	9.6	1684	1332	12.22	13.91	16.34	19.78	12.13	13.86	91.43	133.69
2	17.61	7.2	11.2	1496	1073	10.96	13.08	13.68	15.47	12.52	13.57	64.24	73.25
27	17.21	8.3	10.3	1636	1249	11.29	12.70	13.24	14.78	11.59	12.40	89.75	87.41
Site class II (14.7 to 16.7 m)													
15	16.63	8.5	11.6	1592	1111	10.72	12.25	13.60	14.94	12.89	15.04	107.33	115.23
17	16.98	9.5	12.6	1651	937	11.93	12.46	19.09	15.99	12.48	13.58	120.58	82.73
25	15.22	19.0	21.0	861	694	14.57	16.06	18.39	19.72	17.31	19.13	155.63	164.17
29	16.41	7.3	9.3	1884	1768	9.87	10.71	12.49	13.98	10.72	11.27	93.26	103.98
Site class III (12.7 to 14.7 m)													
3	14.65	8.2	12.2	2289	1929	8.73	10.11	11.40	13.55	8.25	9.50	49.00	65.51
5	14.04	11.2	14.2	2423	2123	10.81	11.77	12.27	13.92	10.90	12.78	125.14	156.71
7*	14.33	17.7	19.1	832	796	11.81	12.02	14.40	14.43	15.12	14.73	78.71	80.04
11	13.90	7.5	10.6	1972	1945	8.47	9.63	11.47	13.01	9.45	9.95	53.14	67.29
13	13.05	8.5	11.6	1567	1361	8.90	9.82	11.24	12.24	9.36	10.14	42.23	48.60
23	13.80	13.5	15.5	1622	569	11.26	11.89	13.51	13.42	11.97	13.73	95.15	45.95
26*	13.42	9.3	11.3	1538	905	11.86	10.96	14.06	13.68	11.65	9.07	86.29	53.88
1	12.89	7.2	11.2	1616	1548	9.79	10.48	11.62	12.61	10.44	11.56	42.82	73.70
28	13.12	8.3	10.3	2500	1692	9.62	10.20	12.30	12.63	9.10	9.57	73.22	67.05
Site class IV (10.7 to 12.7 m)													
8	12.29	23.7	27.7	476	476	16.60	18.49	21.26	22.78	27.90	29.47	236.99	299.50
14	11.86	7.5	10.6	1924	1347	8.17	8.93	9.35	10.64	9.30	10.24	45.21	43.91
22	11.89	8.5	10.5	2046	1228	8.56	9.39	11.43	11.62	10.11	10.09	67.03	51.33
12	10.91	30.5	33.4	356	342	15.79	16.65	18.52	19.35	28.80	29.84	177.70	193.37
Site class V (8.7 to 10.7 m)													
4	10.12	3.2	7.2	1850	1650	6.97	9.08	9.14	9.90	5.76	8.30	11.46	32.60
20*	9.81	7.5	8.7	2632	2179	6.35	6.62	8.71	9.00	6.47	6. 68	20.28	19.38
24	8.65	29.0	31.0	608	608	14.84	16.52	19.86	21.10	22.26	23.23	181.80	214.75
30	9.87	9.3	11.3	1560	910	7.88	8.64	9.78	9.94	9.13	9.94	31.86	23.78
Min	**8.65**	**3.2**	**7.2**	**356**	**342**	**6.35**	**6.62**	**8.71**	**9.00**	**5.76**	**6.68**	**11.46**	**23.78**
Max	**18.68**	**30.5**	**33.4**	**2632**	**2502**	**16.60**	**18.49**	**21.26**	**22.78**	**28.80**	**29.84**	**236.99**	**299.50**

*Plots left out in developing diameter and volume yield growth functions; **at base age 15 years.

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where, *a, b, c, d, e* are the parameters to be estimated; *H* is the top height at age *t*; *S* is the site-index; $d = \ln[H_1/(aH_1^b)]/\ln[1\text{-}\exp(-ct_1)]$; t_1, t_2 = ages (years) at periods 1 and 2; and H_1, H_2 = heights at t_1 and t_2 respectively.

Equation 1 is a base-age variant model and has the desirable sigmoid and asymptotic property of a biological growth function. Models 2 and 3 are well-behaved base-age invariant functions, which can be used to estimate height at any age given the height at any other age. Although models 2 and 3 are base-age invariant, their prediction accuracy may vary depending on the predicted age t_1. To estimate the parameters in models 2 and 3, we have used only two adjacent heights, i.e. each height is predicted by the height one interval away from it. This significantly reduces the sample size and computational time required.

Diameter and volume

Diameter is a convenient variable for predicting the growth of trees in the plantations and forests. Being closely correlated with stem volume, diameter is easy to measure and is an essential quantity for economic and silvicultural decision-making. Quadratic mean diameter is an important element of a stand growth model and is essential for estimating product yield. This may be derived from the per-hectare basal area and number of stems per hectare.

Estimates of volume yield are an important part of a stand model. In an even-aged plantation, the available timber volume at a given age may be estimated using an empirical yield function, which is based on observations of growing stock volumes at various development stages. Yield function is a frequently used database for regional timber resource forecasting and estimates the production potential for a discrete number of site quality classes. Here, stand volume has been modelled against stand age where model parameters are given as a function of stand density and site class.

The following are some of the most frequently used modifications of the exponential function in forestry:

Schumacher (1939):

$$y = a\exp\left(\frac{-b}{x}\right) \qquad (4)$$

Gompertz (Medawar 1940, Yang *et al.* 1978):

$$y = a\exp[-b\exp(-cx)] \qquad (5)$$

Chapman–Richards (Richards 1959, Chapman 1961):

$$y = a[1-\exp(-bx)]^c \qquad (6)$$

where y is diameter or volume and x represents age.

These growth curves were fitted to the data to develop diameter–age and volume–age functions. A serial correlation is often encountered in repeated measurements of the same plots. Hence a 2-stage analysis was considered appropriate in the present case. For fitting the growth curves, first the data from the sample plots were separated on the basis of site class, then within each site class the data were grouped into different stand densities. After that the above functions were applied to these grouped data sets in each of the site classes and, based on the fit statistics (R^2, RMSE and significance of parameter estimates), the best model for developing growth and yield functions was selected. The parameters of the selected function were modelled against the stand densities in each site class and equations were generated. Further parameter estimates of these equations were modelled against different site classes to get the relationship between the parameters and site classes.

Results and discussion

The data given in Table 1 show that the tallest dominant tree recorded was 22.78 m at the age of 27.7 years (plot No. 8). At the age of 11.3 years, the best site (site class I) recorded the dominant height as high as 18.48 m (plot No. 10) while on the poorest site (site class V) it was only 9.94 m (plot No. 30). Also, at the age of 8.3 years the dominant height was 12.30 m (plot No. 28) on a poor site (site class III) and 18.21 m (plot No. 10) on the best site (site class I). The maximum average diameter monitored in the trees was 29.84 cm at the age of 33.4 years (plot No. 12). All the three PSPs exceeding 20 cm average dbh at the final age came from stands with low values of stocking. Of the three plots, which were 25 years old or older, two had volume yield of 190–215 $m^3 ha^{-1}$ and one was about 300 $m^3 ha^{-1}$. The range of initial stocking varied between 356 and 2632 stems ha^{-1}.

A decrease in standing basal area and volume yield was observed in some of the plots because of tree mortality due to various reasons. Measurements in plots Nos. 6, 7 and 20, which were laid out in the first and second year of the experiment, could not be taken after two years as the plantations were submerged in water due to waterlogging and hence these plots were abandoned. In plot No. 26 a large decrease was observed in standing basal area as well as volume yield due to heavy mortality of the bigger trees. Therefore these four plots were left out and the remaining 26 plots were considered in the study.

Height

The data collected from the sample plots showed that the growth in the average height of the trees was observed as high as 1.68 m $year^{-1}$ on the best site (site class I, plot No. 10) while it varied from 76 cm to 1 m $year^{-1}$ (plots No. 14, 20, 22 and 30) for poorer sites (site classes IV and V) in the plantations of 8 to 12 years of age. During the initial 5 years of growth, height growth as high as 2.18 m $year^{-1}$ (plot No. 4) was recorded. This compares well with earlier observations reported in the literature that on good sites a height growth of 1.52 m $year^{-1}$ may be maintained for the first 10 years while on poorer sites this is recorded as between 0.91 and 1.22 m $year^{-1}$ (Bakshi 1941). Site index, a measure of site quality, is the height of the dominant or co-dominant and healthy trees at a predetermined age referred to as base or index age, which is somewhat less than rotation age. In the present study, the base age was selected as 15 years and the site index was estimated for each plot at this base age using the dominant height and age data collected for the particular plot. The site index thus obtained for the plots varied between 8.65 and 18.68 m. Accordingly, five site classes were defined at 2-m intervals.

Table 2 presents the values of estimated parameters and fit statistics obtained by fitting various site index equations to the dominant height and age data set. Although R^2 measures the goodness of fit, i.e. the percentage of total variation in the data accounted for by each model, it does not provide information about the relative accuracy of the fitted models because of the differences in data structures used to fit the models. Hence root mean square errors and average bias estimated for the three fitted models were used. Table 2 shows that the function with the minimum RMSE and bias and maximum R^2 is the Goelz–Burk function.

Figure 1(a) presents the performance of the models 1, 2 and 3 in predicting dominant tree heights in the plots. The models were fitted against the measured dominant height of the trees in all the 30 plots distributed among the five different site classes. The fluctuation observed in the figure for the dominant height of the trees is due to the fact that the dominant height of the trees was large at the better site while it was low at the poorer sites, which is obvious. The figure indicates that the performance of model 2 was the best closely followed by model 3. Hence model 2 was used to develop dominant height vs. age curves for different site classes and the same are shown in Figure 1(b).

Table 2 Parameter estimates and fit statistics for dominant height polymorphic models

Model	N	Parameter	Estimated value	Asymptotic standard error	R^2	RMSE	Bias
Ek	103	a	41.92043	7.74054	0.769	1.60	-0.0996
		b	-0.07046	0.04929			
		c	0.01554	0.00299			
		d	3.21775	0.32650			
		e	-0.71263	0.38011			
Goelz–Burk	74	a	0.000036	0.000013	0.975	0.52	0.0765
		b	-7.40080	1.45534			
		c	1.66450	0.38189			
		d	0.24775	0.03863			
Payandeh–Wang	74	a	1.59174	0.58687	0.972	0.55	0.1590
		b	0.84075	0.12692			
		c	0.62785	0.07157			

Figures 2 (a)–(c) show the plot of residuals obtained by fitting the models 1, 2 and 3 respectively on the dominant height data set of the *D. sissoo* plots. A trend line (regression line based on two-period moving average) is also added in Figure 2. The residuals varied from -3.6022 to 3.1072 for model 1, from - 0.7905 to 1.3669 for model 2, and from - 0.8792 to 1.4156 for model 3. The error structure indicates that model 2 fitted well to the data set. Model 3 appears to be placed second.

Diameter

During 8–12 years of age, the diameter growth as high as 1.6 cm year^{-1} (plot No. 10) was maintained on the best site (site class I) while on poorest site (site class V) it ranged from 0.8 to 1.0 cm year^{-1} (plots No. 20 and 30). During the first five years of growth, diameter growth as high as 1.8 cm year^{-1} (plot No. 4) was observed. This is in close conformity with the reports available in the literature that diameter growth of 1.5 cm year^{-1} may be maintained for the first 10 years while on poorer sites it is reduced to 0.7 and 1.1 cm year^{-1} (Bakshi 1941). Equations 4–6 were fitted on the diameter and age data sets. The method adopted is described in the last paragraph of the previous section. R^2 varied from 0.827 to 0.999, from 0.860 to 0.999, and from 0.903 to 0.999, while RMSE varied from 0.0127 to 0.3514, from 0.0109 to 0. 4159, and from 0.0099 to 0.3629 for the Shumacher, Chapman–Richards and Gompertz functions respectively. On the basis of fit

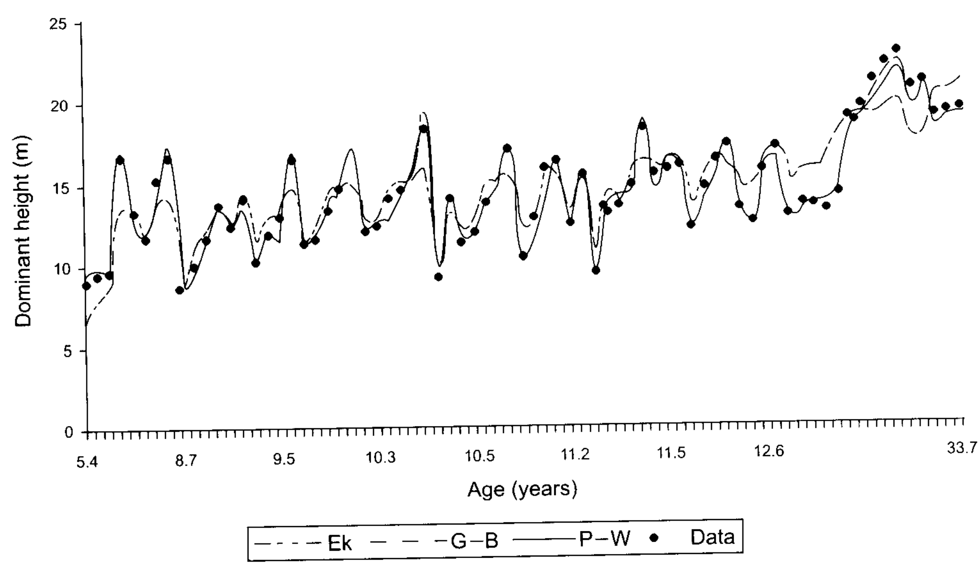

Figure 1(a) Performance of the three height models (lines) against the measured dominant heights of the trees in 30 *Dalbergia sissoo* plots spread over five different site classes

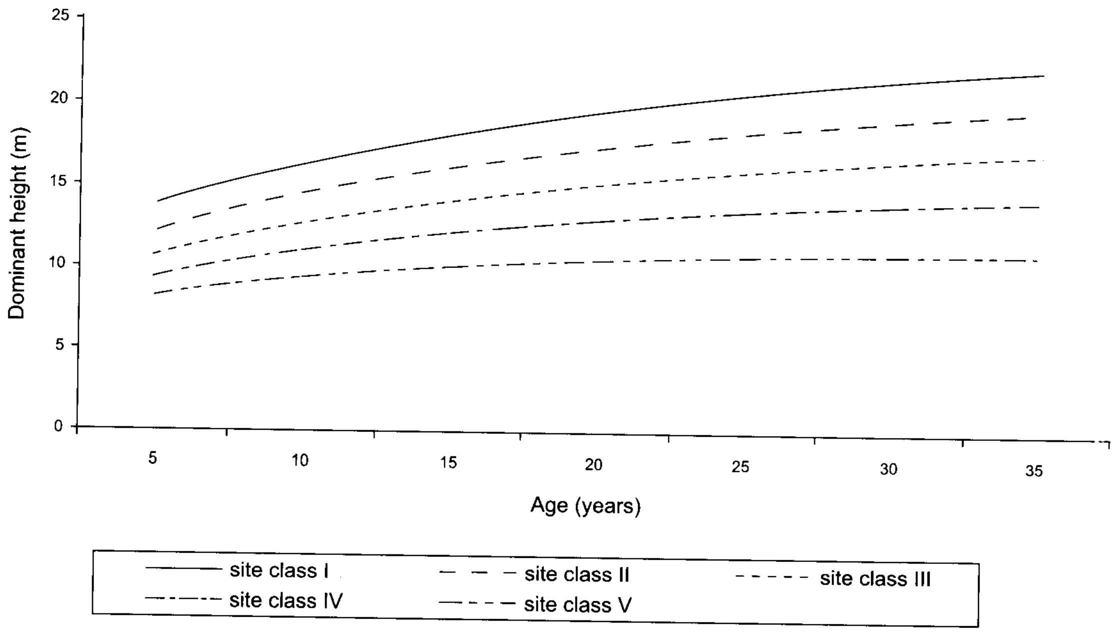

Figure 1(b) Height growth curves for different site classes

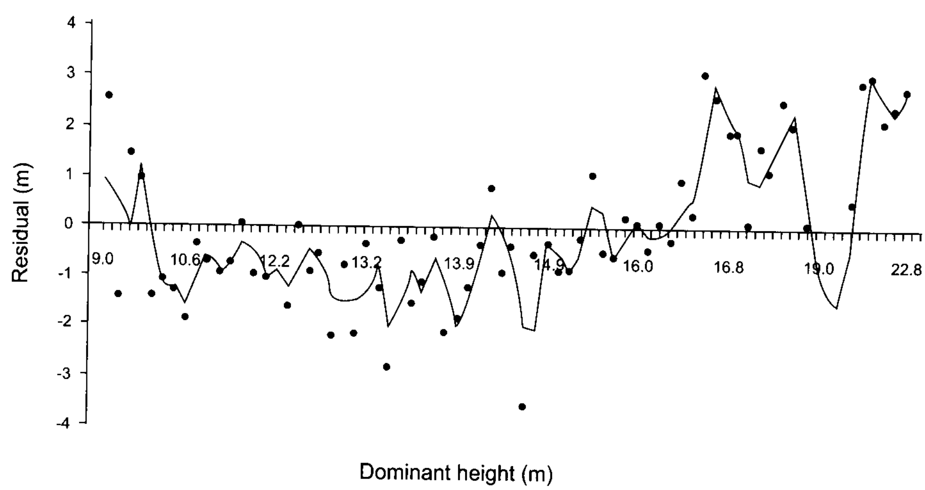

Figure 2 Residuals for dominant height obtained by fitting the models on the data set of *D. sissoo* plots (along with the trend line): **(a)** Ek model

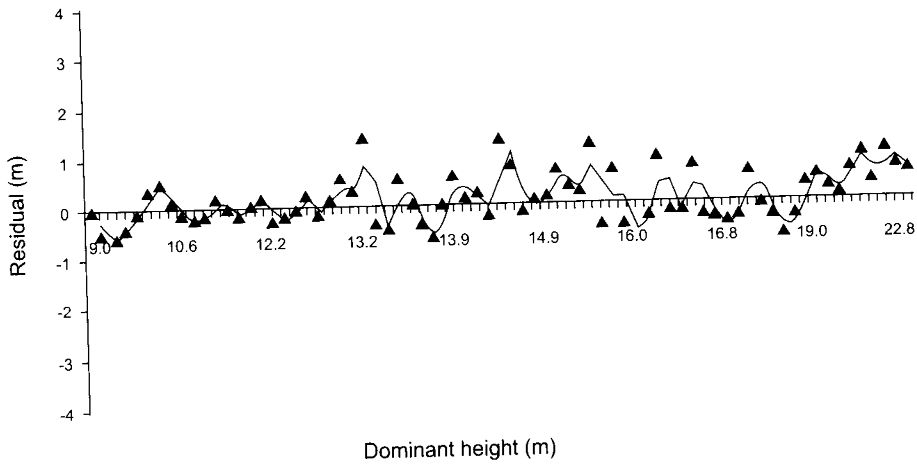

Figure 2(b) Goelz–Burk model

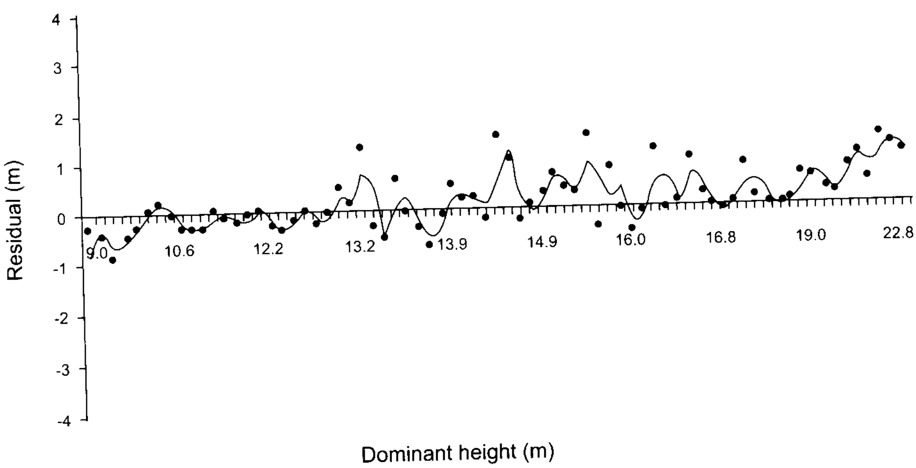

Figure 2(c) Payandeh–Wang model

statistics (R^2, RMSE) it was found that the Gompertz function performed best among these functions. The results may be generalized as follows:

Gompertz function

$$D = a\exp[-b\exp(-cA)] \tag{7}$$

In the above equation, D is the diameter and A is the age. The parameters a, b and c of the equation were modelled against the stand densities and the equations of the following form produced the best results:

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$$a = a_1 + b_1 Den + c_1 Den^2 + d_1 Den^3$$

$$b = a_2 + b_2 Den + c_2 Den^2 + d_2 Den^3 \qquad (8)$$

$$c = a_3 + b_3 Den + c_3 Den^2 + d_3 Den^3$$

In the above equations, *Den* is the stand density. A step-wise regression technique was adopted for modelling and only the variables giving best results were retained. In this process we found that for

$$\begin{array}{ll} \textit{site classes I, IV \& V:} & d_1,\ d_2 \text{ and } d_3 = 0 \\ \textit{site class III:} & b_1,\ d_2 \text{ and } c_3 = 0 \\ \textit{site class II:} & d_1,\ b_2 \text{ and } b_3 = 0 \end{array} \qquad (9)$$

(for site class III, d_1 = 1.45E-08 and for site class II, d_2 = 1.41E-08)

Further, the parameters a_i, b_i, c_i and d_i (i = 1 to 3) were modelled against the site indices using a curve estimation procedure and the following quadratic and cubic equations were obtained:

$$\begin{array}{ll} a_1 = -692.2301 + 117.3150xSI - 4.5455xSI^2 & R^2=0.994 \\ a_1 = 0.5275 - 0.0095xSI^2 + 0.0005xSI^2 & R^2=0.967 \\ c_1 = 0.0007 + 1.0829E - 05xSI^2 - 4.7126E - 07xSI^3 & R^2=0.865 \\ a_2 = 12586.3040 + 1563.8864xSI - 47.4241xSI^2 & R^2=0.721 \\ b_2 = 47.8968 - 6.0339xSI + 0.1847xSI^2 & R^2=0.801 \\ c_2 = -0.0200 + 0.0025xSI - 7.7104E - 05xSI^2 & R^2=0.730 \\ a_3 = 97.8799 - 14.5548xSI + 0.513xSI^2 & R^2=0.933 \\ b_3 = -0.1018 + 0.0153xSI - 0.0006xSI^2 & R^2=0.995 \\ c_3 = 5.7658E - 05 - 8.4510E - 06xSI + 2.9281E - 05xSI^2 & R^2=0.996 \\ d_3 = -7.9864E - 09 + 5.1088E - 10xSI & R^2=0.966 \end{array} \qquad (10)$$

In the above equations, *SI* represents the site index. The curves representing development of mean diameter against the stand age for *D. sissoo* plantation (1600 stems ha^{-1}) for different site classes are shown in Figure 3. The figure shows that the diameter growth was much higher for sites I and II compared with sites III, IV and V.

Volume growth ranged from 2.10 $m^3\,ha^{-1}\,year^{-1}$ (site class V, plot No. 30) to 19.90 $m^3\,ha^{-1}\,year^{-1}$ (site class I, plot No. 10) depending upon age, density and site quality. Volume growth ranging from 1.05 and 10.92 $m^3\,ha^{-1}\,year^{-1}$ is reported from Pakistan according to site quality for 5–20 years of age (Bakshi 1941).

In the present study, *D. sissoo* planted at 2 x 2 m spacing produced volume yields of 5.98 to 15.25 $m^3\,ha^{-1}\,year^{-1}$ at the age of 8.3 years according to site quality. At a spacing of 2 x 3 m, the plantations of age 8.6 years produced MAI of 2.23 $m^3\,ha^{-1}\,year^{-1}$ on the poorest site while on the best site it was 16.04 $m^3\,ha^{-1}\,year^{-1}$. MAI ranging from 4.43 to 8.50 $m^3\,ha^{-1}\,year^{-1}$ at the age of 10 years is reported from the plantation in Bihar State of India planted at the spacing of 2 x 2 m (Trivedi 1986).

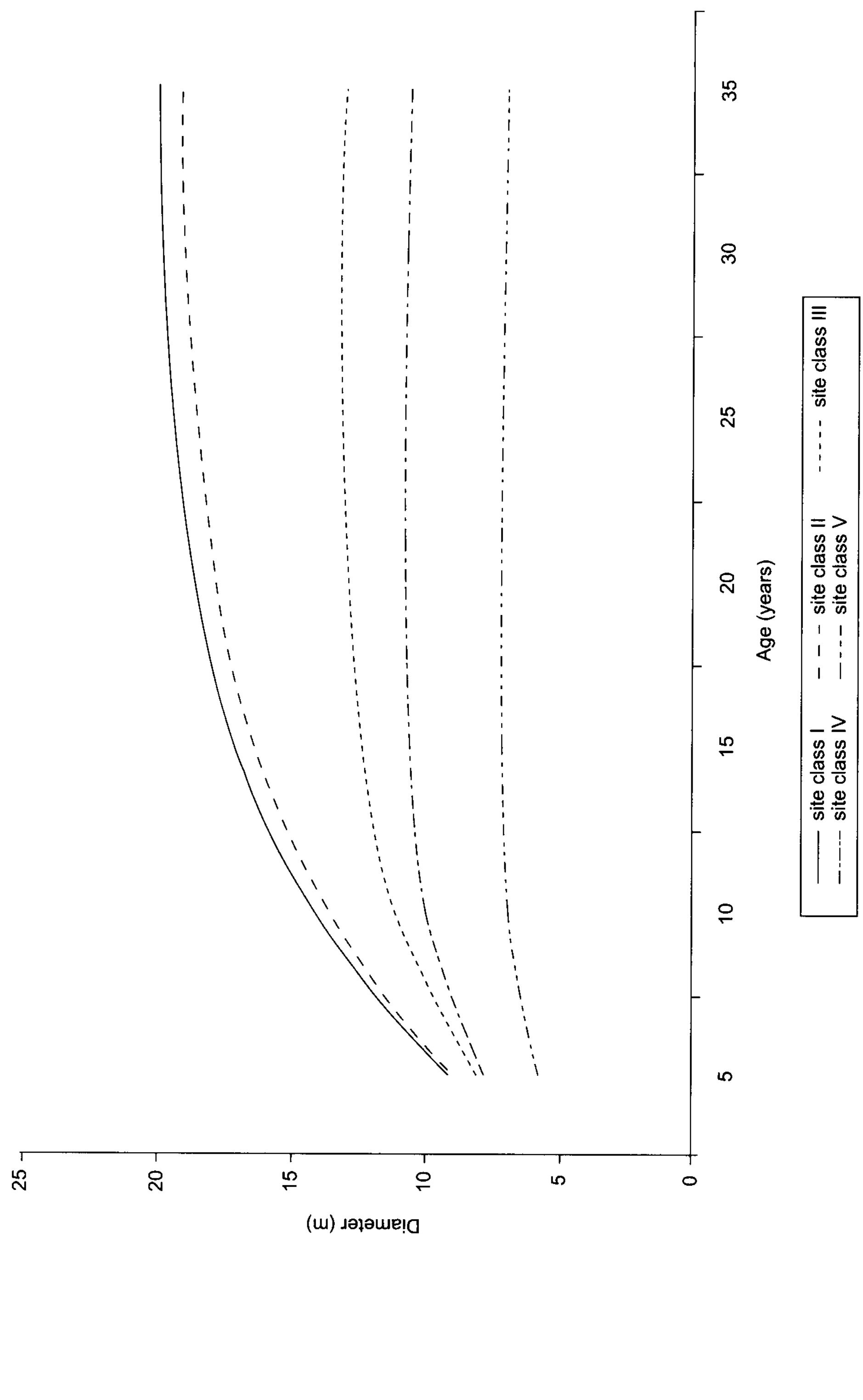

Figure 3 Development of mean diameter against stand age for *D. sissoo* plantations for different site classes

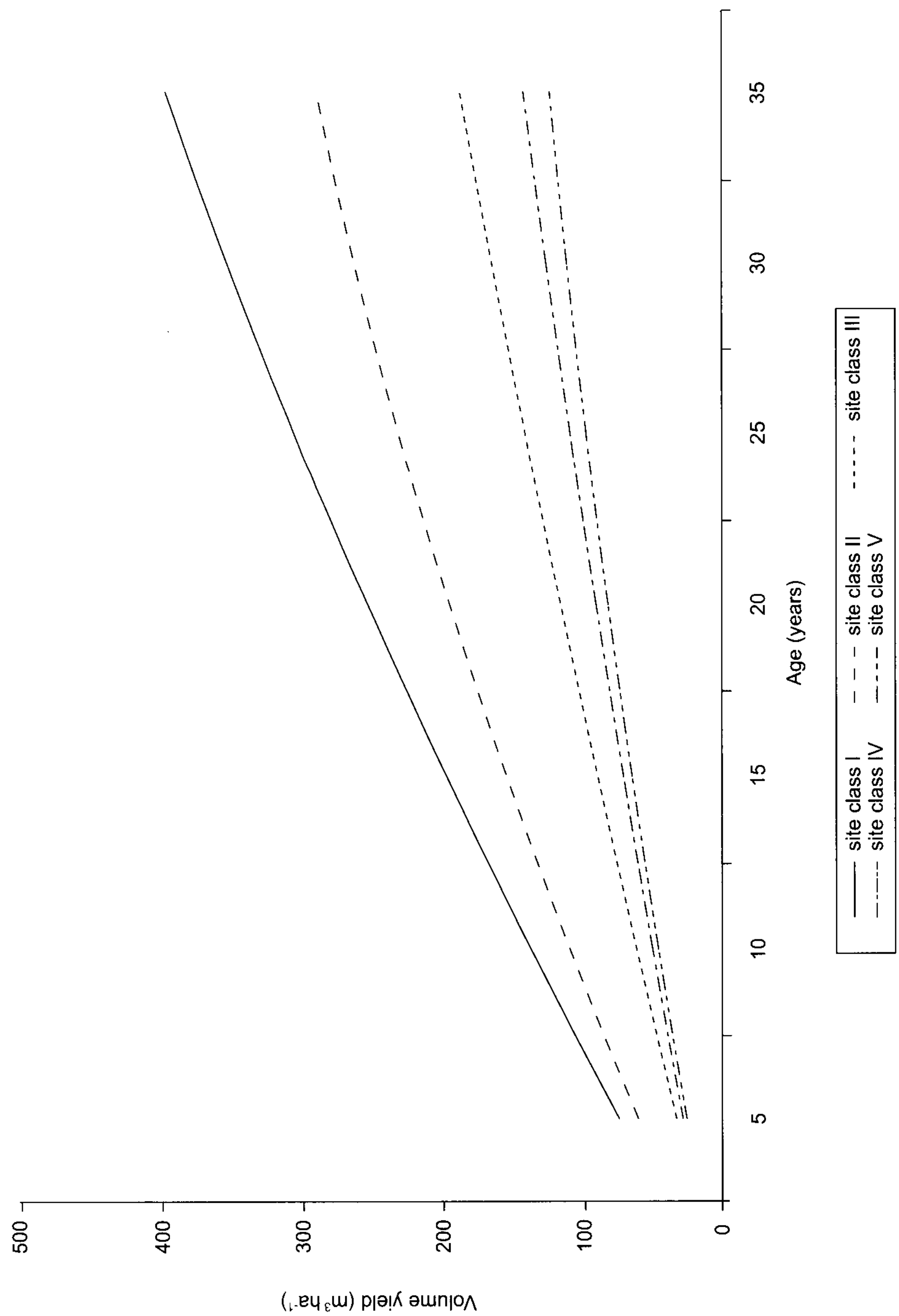

Figure 4 Volume yield vs. stand age curves for *D. sissoo* plantations for five different site classes

The ratio of the best to the worst volume yield at the age of 8.5 years with 2 x 2 m spacing was found to be about 7:1.

All the three equations, viz. Schumacher, Chapman–Richards and Gompertz functions, were also fitted on the data sets of volume yield (V) and age as per the method described earlier. R^2 varied from 0.782 to 0.998, from 0.910 to 0.999, and from 0.915 to 0.999, while RMSE varied from 0.9455 to 6.1772, from 0.4889 to 4.8477, and from 0.4889 to 4.7965 for the Shumacher, Chapman–Richards and Gompertz functions respectively. Based on the fit statistics, it was found that the Gompertz function performed better in comparison with the other two functions. The result may be summarized as follows:

Gompertz equation:

$$V = a \exp[-b \exp(-cA)] \qquad (11)$$

The parameters *a, b* and *c* were modelled against the stand densities and the equations of the following form produced the best results:

$$a = a_1 + b_1 Den + c_1 Den^2$$

$$b = a_2 + b_2 Den + c_2 Den^2 \qquad (12)$$

$$c = a_3 + b_3 Den + c_3 Den^2$$

Further, the parameters a_i, b_i, and c_i (i = 1 to 3) were modelled against the site indices using curve estimation procedure and the following quadratic and cubic equations were obtained:

$$a_1 = -7513.1362 + 168.7586xSI - 7.9210xSI^3 \qquad R^2=0.999$$
$$b_1 = 1.9384 - 0.1244xSI^2 + 0.00064xSI^3 \qquad R^2=0.970$$
$$c_1 = 0.0014 + 1.5397E-05xSI^2 - 1.0605E-06xSI^3 \qquad R^2=0.932$$
$$a_2 = -42304.2539 + 4893.8935xSI - 139.6208xSI^2 \qquad R^2=0.980$$
$$b_2 = 134.1936 - 15.688xSI + 0.4537xSI^2 \qquad R^2=0.985 \qquad (13)$$
$$c_2 = -0.6271 + 0.0074xSI - 0.0002E-05xSI^2 \qquad R^2=0.986$$
$$a_3 = -20.6505 + 2.0960xSI - 0.0026xSI^3 \qquad R^2=0.754$$
$$b_3 = 0.0115 - 0.0009xSI + 6.4869E-07xSI^3 \qquad R^2=0.913$$
$$c_3 = -1.5165E-05 + 2.0599E-06xSI - 6.4969E-08xSI^2 \qquad R^2=0.969$$

The stand volume over age curves for *D. sissoo* plantation (density 1600 stems ha^{-1}) for different site classes are shown in Figure 4. The figure indicates that the growth in volume yield is much higher for sites I and II in comparison with sites III, IV and V.

Conclusions

Error structure has a large impact in model selection for development of site-index equations, especially when it includes testing significance of parameters. The parameters of a difference equation (e.g. model 2) will depend on which differences are used to fit the equation (previous measurement vs. all possible differences). In the present study, the differences of consecutive management produced better result in comparison with all possible differences.

Height growth models often predict heights at an age based on height at another age (site index). By fitting base-age invariant site index equations, site index and height prediction equations are fitted together. It is known that individual equations will have lower variance (Curtis *et al.* 1974) but in that case neither the height prediction equation nor the site index equation will possess a shape that represents the true relationship between height and age across levels of site-index (Goelz & Burk 1992). For example, when predicting height based on site index, the predicted height at the base age will be overestimated for a low site index and underestimated for a high site index. This behavior may be avoided to some extent by fitting difference equations as is evident from Figure 2. The choice of predictor age influences the models, accuracy and bias. The choice of base age is typically specified so that it is less than the rotation age.

The Goelz–Burk model, which is based on the algebraic difference form of the Chapman-Richards function, produced the best fit for the development of the top height model/site index equation. The Gompertz function was found suitable for the prediction of diameter growth and volume yield per hectare at any time in the study area. The model includes the effect of stand density and site quality on the diameter and volume growth as the coefficients of the model have been fitted as functions of the density and site class.

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