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OPTIMUM ROTATION AGE OF CHINESE FIR (*CUNNINGHAMIA LANCEOLATE*) PLANTATIONS UNDER FAUSTMANN AND MAXIMUM SUSTAINABLE YIELD CRITERIA

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Most forest research focuses either on the silvicultural aspects of forest management or the economic optimisation of an objective function to answer a specific policy question. This paper investigated optimum rotation ages for harvest, under both economic and biological theories, to discover the conditions that maximise both profit and sustainability. Both the optimum biological rotation for maximising sustainable yields (MSY), and economic rotation based on the Faustmann model were calculated through observation data from Chinese fir (*Cunninghamia lanceolata*) plantations in Fenyi, Jiangxi Province, China. It was found that the economic rotations of Chinese fir were longer than the biological rotations by almost 2 years when the discount rate equaled 0.05. If the discount rate increased as high as the inverse of the biological rotation time plus the incremental growth rate of the stumpage price, the rotations under both Faustmann and MSY rules coincided. Under these conditions, the ratio of regeneration costs to the value of timber in a stand should be low to keep forest management profitable. These findings are of practical importance for forest managers in relation to forest investment, planting decisions and harvesting for bussiness.

Keywords: Faustmann criterion, maximising sustainable yields (MSY), Chinese fir plantations, forest management

INTRODUCTION

Developing optimal harvest schedules is important for meeting the objectives of landowners or forest managers (Bettinger & Tang 2015). A common way to meet landowners' objectives is to maximise their perceived benefit. Efforts to give forestry a rational economic basis are almost as old as regulated forestry itself. Economists view the timber production process as "growing capital" and examine how the earned income and the production costs differ depending on the age of the stand, based on the economic theory that the maximum benefit is reached when the marginal benefits equal the marginal costs (Lobovikov & Nga 2019). The Faustmann model is one of the most widely used metrics to quantify the perceived benefits for landowners, and it often represents the decision makers' preference or goal in the optimisation system. The optimum rotation age using the Faustmann model with different levels of complexity has been studied by various researchers (Hartman 1976, Reed 1993, Tahvonen et al. 2001, Price 2017, Brazee 2018, Chang 2020). However, the Faustmann model simply defines an equilibrium harvest age that would produce an economic yield, as opposed to the maximum sustainable yield (MSY).

The MSY implies a balance between growth and harvest potential with harvest at an age where average annual growth of timber volume is maximised, resulting in the largest harvest level that does not deplete the future timber resource (Luckert & Williamson 2005, Olschewski & Benitez 2010). Forest managers consider the decision as reasonable. Under the MSY criteria, researchers have focused on timber yield and biological productivity, leaving the economic factors unexplored (Battaglia & Sands 1998, Ivanova 2017). The MSY approach generally does not consider the value of the standing timber or management costs. Therefore, the MSY approach ignores the implications of economic efficiency for management decisions (Newman et al. 1985). For this reason, it is often called the "forestry" decision (Kaya et al. 2016).

Under the MSY approach, it is necessary to determine the optimum rotation age and estimate the amount of wood produced at this age. In contrast, the Faustmann approach describes the yield as a function of input factors which enable forest managers to study the effects of management decisions on timber yield, and the relationships among the various input factors. Such information could be very helpful to guide forest managers in resource allocation decisions (Nautiyal & Couto 1982). Few studies take both MSY and Faustmann criteria into consideration to determine the optimum rotation age of a stand of trees. Gaffney (1960) proved that only when the discount rate in the Faustmann model approached zero, did the optimal economic rotation approached MSY rotation age. Binkley (1985) found that the Faustmann and MSY rotation ages were similar only when regeneration and management costs were ignored. Even if a very simple cost formulation was introduced into the problem, the Faustmann rotations equaled or exceeded the MSY rotation. Dykstra (1986) showed that when unit logging costs decline with tree size, the optimal economic rotation can exceed the MSY rotation.

The relationship between Faustmann and MSY rotation age is still ambiguous. Gaffney's result assumed that the regeneration cost was constant, while Binkley and Dykstra's results assumed a variable cost. This suggests that a problem may exist whereby the results are particularly sensitive to the choice of methodological assumptions. A more realistic model of forest management would relax the assumptions of the analysis because forest managers make choices with respect to site type, planting density and stumpage prices when the economic environment changes. This will result in different growth, cost and income curves and ultimately different rotations. Therefore, we need information to provide insights into whether silvicultural activities, such as initial planting density, are a wise use of capital. In addition, to satisfy both profitability and sustainability, the impact of stumpage price and discount rate should be explored.

This paper derived mathematical and empirical estimates of returns to provide insights into analysing optimum rotations in the context of both MSY and Faustmann criteria to facilitate better forest management. The theoretical MSY and Faustmann framework of rotation was analysed mathematically. Chinese fir was used as an example to assess the optimum rotation ages for the different initial planting densities and discount rates.

The maximum sustainable yield function and rotation

If the management objective is to maximise long-term yield, the tree or stand would be harvested at the age when the mean annual increment (MAI) and the current annual increment (CAI) cross (Bettinger & Tang 2015). The MAI refers to the average growth per year that a tree or stand of trees has exhibited to a specified age and is calculated as the growth equation divided by time:

$$t\text{—MAI} = Q(t)\ /t$$

where $Q(t)$ = yield at time t. The CAI refers to the growth for one specific year or any other specified period and is calculated as the first derivative of the growth equation (Q_t) (Bettinger & Tang 2015). The point where the marginal increase in yield is equal to the average yield of the stand is typically referred to as the biological rotation age and defines the potential productivity of a stand of trees. When MAI = CAI, we get $Q_t/Q = 1/t$. Note that the interest rate has dropped out, impling that no substitution possibilities exist between land and capital (Jacobsen et al. 2018).

The typical growth pattern of most trees over time approximates to a sigmoid curve. The yield of a stand of trees at time t [$Q(t)$], has been studied for decades, and most of the proposed mathematical functions deal with the relationship between stand volume and age. The study of the relationship between growth, stand age and initial planting density has resulted in very few proposed mathematical functions. Growth and yield expectations under changing silvicultural management are still a focus of scientific investigation. Mills and Jackson (1980) proposed a two variable density growth and yield function. Nautiyal and Couto (1982) examined several possible yield functions and expressed the yield function as $Q(t, m) = Ae^{\alpha/t+\beta m^2}$, where A, α and β are coefficients and $Q(t, m)$, t, and m are stand volume, stand age and initial planting density, respectively. In contrast to Mills and Jackson model, Nautiyal and Couto's equation passed through the origin and was quasi-concave. Yet, all CAI curves peaked at the same age regardless of initial planting density and all stands grew at the same rate (Chang 1984). Chang (1984) improved previous models by allowing the CAIs to reach their peaks at different ages. Chang's function is expressed as

$$Q(t, m) = Ae^{\alpha/t^2 + \beta/mt}$$

Coefficient A represents the theoretical maximum volume or the biological carrying capacity of the site. The expression e^{α/t^2} defines the curvature and the inflection point of the yield curve when the planting density approaches infinity. This base yield curve is modified by the term $e^{\beta/tm}$ as the planting density decreases. Coefficient A is greater than 0 and both α and β are smaller than 0. Chang's model regarded t, t^2 and the interaction of t and m as independent variables. The initial planting density, m, which is widely considered by forest scientists to impact critically on stand growth dynamics, was not included in the model as an independent variable. However, another term of m was included in the suggested model which regressed the log transformed equation. Thus, the basic model used in this paper was:

$$Q(t, m) = Ae^{\alpha/t^2 + \beta/tm + \gamma/t + \delta/m}$$

Using the derivation Q(t, m) with respect to t, the current annual increment can be expressed as:

$$CAI = \frac{\partial Q(t, m)}{\partial t} = \left(\frac{-2\alpha}{t^3} - \frac{\beta}{t^2 m} - \frac{\gamma}{t^2}\right) Ae^{\alpha/t^2 + \beta/tm + \gamma/t + \delta/m} \quad (1)$$

The mean annual increment can be expressed as:

$$MAI = \frac{Q(t, m)}{t} = \frac{1}{t} Ae^{\alpha/t^2 + \beta/tm + \gamma/t + \delta/m} \quad (2)$$

When CAI = MAI, $t^{*2} + \left(\frac{\beta}{m} + \gamma\right) t^* + 2\alpha = 0$.

If α, β and γ are smaller than 0, $t > 0$:

$$t^* = \frac{-(\beta/m + \gamma) + \sqrt{(\beta/m + \gamma)^2 - 8\alpha}}{2} \quad (3)$$

Thus, the larger the m is, the smaller the t^* will be.

Faustmann model and optimum economic rotation

Forests may be managed to return the greatest revenue, rather than for sustainability. Forest economists calculate the cutting age which generates the maximum profit. The Faustmann formula is commonly used to estimate the land expectation value of bare land where trees are simultaneously planted, felled and replanted, and the processes repeated indefinitely (Bettinger & Tang 2015). Under these assumptions, Faustmann's model aims to get the maximum profit by calculating the forest land expected value (LEV) by discounting the value of future revenue. The LEV is expressed according to Cairns (2017):

$$LEV = \frac{pQe^{-rt} - c}{1 - e^{-rt}} \quad (4)$$

where Q denotes the standing timber volume, which varies with the final harvesting age (t) and the initial planting density (m), c denotes the regeneration cost, including site preparation, digging pits, seedling cost and competitive vegetation control etc., p is the stumpage price and r is the discount rate. The management goal is to maximise the net present value (NPV) of the forest land. If forest management is profitable, the term $pQe^{-rt} - c$ should be larger than 0, thus, $c/pQ <= e^{-rt}$. The optimum economic rotation is the age t that makes the first-order condition of LEV equal 0. The first-order condition can be expressed by:

$$\frac{Qt}{Q(t) - c/p} = \frac{r}{1 - e^{-rt}} \quad (5)$$

This expression is also shown in previous research which assumes that the stumpage price, p, is constant along with stand age, t (Binkley 1985, Binkley 1987, Jacobsen et al. 2018). Under this assumption, Gaffney (1960) reported that the optimum economic rotation would be shortened, while Newman et al. (1985) reported that rotations would be too long, with large losses in NPV. The constant price assumption was relaxed and the optimum economic rotations were analysed under the condition that the stumpage price was dependent on stand age. The first-order condition was be expressed by:

$$\frac{Q_t}{Q} = \frac{r - cr/pQ}{1 - e^{-rt}} - \frac{Pt}{p} = \frac{r}{1 - e^{-rt}} \left(1 - \frac{c}{pQ}\right) \leq \frac{r}{1 - e^{-rt}} \quad (6)$$

where, Q_t/Q is the growth rate of Q, denoted by QG, p_t/p is the growth rate of p, denoted by PG, c/pQ is the benefit cost ratio, denoted by CR, assumed $r/(1\text{-}e^{-rt}) = A$ and QG = (1-CR)A-PG. The optimum economic rotation time (t^{**}) is the point at which the curve of QG intersects with the curve of (1-CR)A-PG. The optimum biological rotation (t^{*}) is the point at which the curve of QG intersects with the curve of 1/t (Figure 1). The t^{**} can be shorter or longer than t^{*}, depending on the discount rate (r), the level of (1-CR) and PG. When the discount rate r is relatively small, mathematically, $r \rightarrow 0$, $\lim\limits_{r \rightarrow 0} \frac{r}{1\text{-}e^{-rt}} = \frac{1}{t} \cdot \frac{r}{1\text{-}e^{-rt}} < \frac{1}{t}$ is obtained.

In Figure 1,the curve of (1-CR)A-PG is below the curve of 1/t, making the intersection point t^{**} fall to the right of t^{*}. When the discount rate is high, the curve of (1-CR)A-PG shifts higher, making t^{**} decrease.

If two dotted curves were to intersect (Figure 1), $(1\text{-}CR)A\text{-}PG = \frac{1}{t}$, $CR = 1\text{-}\frac{1\text{-}e^{-rt}}{r}\left(\frac{1}{t}+\frac{P_t}{p}\right)$ can be obtained, and at the same time, $CR \leq e^{-rt}$ should be satisfied to ensure that forest management is profitable. Thus, the discount rate r should be equal to or less than (1/t + PG). When r is as high as (1/t + PG), the two dotted curves will intersect at the point of CR equal $e^{-(1+t.PG)}$. Unless subsidised, no production is ecpected for these combinations of CR and r. Thus, when the discount rate is high, the forest owner should decrease costs.

From equation (1), we also see that t^{*} will change along with the initial planting density, m. When m becomes higher, t^{*} becomes shorter. In stands with different initial planting density, the relationship between t^{*} and t^{**} will be different. Table 1 shows the relationship between t^{*} and t^{**} under eight conditions. In conditions (2a) and (3b), t^{*} and t^{**} will change in the same direction, and the distance between them is hard to determine. In conditions (1a), (1b), (4a) and (4b), the impact directions of r and CR on t^{**} are the opposite, and a larger impact will determine the change of direction of t^{**}. Thus, long-term observation data is needed to indicate the effect on optimal economic rotations in these six situations. In condition (3a), the distance between t^{*} and t^{**} will be longer. The objectives of maximum profits and sustainable yield will not be satisfied at the same time. Thus, when managing forest under a lower discount rate and higher ratio of CR, higher planting density should be avoided. Finally in condition (2b), t^{*} and t^{**} move towards each other, and they will be superimposed when $r = (1/t + p_t/p)$ and CR < $e^{-(1+t\cdot PG)}$. The calculation of this period is necessary to achieve the economic and sustainability goals of the landowners. If $r > (1/t + p_t/p)$ and CR < $e^{-(1+t\cdot PG)}$, t^{*} will be longer than t^{**}. Table 1 confirms that when the discount rate is high and forest management has low CR, forest owners should use low initial planting densities.

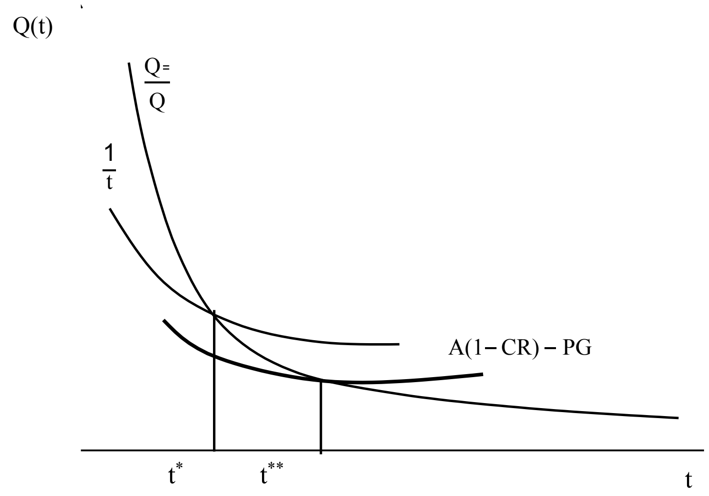

Figure 1 Optimal biological rotation (t^{*}) and economical rotation (t^{**}) for timber management, the optimum economic rotation time (t^{**}) is the point at which the curve of QG intersects with the curve of $(1\text{-}CR)\cdot r/(1\text{-}e^{-rt})\text{-}PG$, the optimum biological rotation (t^{*}) is the point at which the curve of QG intersects with the curve of 1/t

Table 1 Possible results under different initial planting density, discount rate and costs

Initial planting density		Higher r		Lower r	
		Higher CR (1)	Lower CR (2)	Higher CR (3)	Lower CR (4)
Higher density	(a)	t^* will be shorter, t^{**}will be longer when CR becomes larger, but shorter when r becomes higher	t^* will be shorter, t^{**} will be shorter,	t^* will be shorter, t^{**} will be longer,	t^* will be shorter, t^{**} will be shorter when CR becomes smaller, but longer when r becomes lower,
Lower density	(b)	t^* will be longer, t^{**} will be longer when CR becomes larger, but shorter when r becomes higher,	t^* will be longer, t^{**} will be shorter,	t^* will be longer, t^{**} will be longer,	t^* will be longer, t^{**} will be shorter when CR becomes smaller, but longer when r becomes lower,

t^* is the optimum biological rotation, t^{**} is the optimum economic rotation, r is the discount rate, CR is the benefit cost ratio

MATERIALS AND METHODS

Sampling plot selection and population quality

Observational data were compiled on the density of 35-year old Chinese fir trees in an experimental plantation. The experimental forest was established in the spring of 1981 using 1-year-old Chinese fir bare root seedlings in Fenyi, Jiangxi Province. The initial planting densities were 2.0 × 3.0 m (1,667 seedlings ha^{-1}), 2.0 × 1.5 m (3,333 seedlings ha^{-1}), 2.0 × 1.0 m (5,000 seedlings ha^{-1}), 1.0 × 1.5 m (6,667 seedlings ha^{-1}) and 1.0 × 1.0 m (10,000 seedlings ha^{-1}) which formed a block. The trees in the plots were numbered, and the following measurements were taken annually from the third year after planting: total tree height (H), diameter at breast height (DBH), crown width within and between rows, and the height up to the base of the lowest live branch. After 10 years, surveys on forest growth conditions were carried out once in every 2 years. A total of 23 continuous observations were recorded up to the end of 2018.

To investigate whether the results obtained with different sampling plots for initial planting densities varied significantly among different samples of the same population, a single-factor ANOVA test at 5% significance level was used, getting an F-statistic significance of less than 0.05, and rejecting the null hypothesis that the stock volume from different initial planting densities have the same average. This allowed to compare timber volumes across different initial planting densities.

Yield function and optimal biological rotations

Knowing how forest trees grow is vital to understanding when to carry out a final harvest and how to manipulate timber yields. Applications of both MSY and the Faustmann approaches require information on the empirical form of a yield function, Q(t). Data on tree age (t), initial planting density (m) and tree volume (Q) were used to estimate the yield function.

Tree volumes (dm^3) of different ages were estimated using the experimental formulae developed by Liu and Tong (1980) for Chinese fir: $Q = 0.00005877042 \times DBH^{1.9699831} \times H^{0.89646157}$. The stand volume per ha was obtained by summing volumes for all trees per year. The growth process requires one to model the change in standing volume of a stand of trees and to determine what conditions of a stand can ensure the greatest volume growth (Sun et al. 2010).

The yield of Chinese fir was estimated including CAI and MAI and volume stock at different ages of the observed stands. Yield functions and optimal biological rotation ages were estimated by regression using SPSS version 10. A 5% level of significance was assigned to define the goodness of fit of the models. The estimated parameters of the selected models were placed into equations 1–3 for calculating optimal biological rotations.

Optimal economic rotations

As mentioned above, the net price of stumpage is an essential factor to calculate the economic optimum rotation age with the Faustmann model. The model used in this paper allowed prices to change over the stand age. As the published price series data for timber in China were of mixed quality, the average stumpage price of a stand was calculated based on the timber diameter classes observed in Jiangxi Province, and the assorted timber outturn and output of stands with different initial planting density according to stand ages. Washburn (1990) gave the estimate procedure of stumpage price using period-average stumpage prices. The predicted stumpage price of stands under five initial planting densities, for Chinese fir, are shown in Figure 2.

The initial planting costs came from two sources: the variable cost of planting equalen to 0.5 yuan multiplied by the number of trees, and the fixed cost, 15000 yuan ha^{-1}. Thus the initial planting costs ranged from 15,833 yuan to 20,000 yuan per hectare, depending on the specific initial planting densities.

The discount rate selection was based on observations by past studies (Brukas et al. 2001, Price 2017, Brazee 2018). A discount rate of 5% was initially selected to show the results of optimum rotation age, profit, harvest volume and CR with the Faustmann model and MSY for Chinese fir (Table 2). The effect of alternative discount rates was also analysed to test the sensitivity of the results.

RESULTS

Yield function and optimal biological rotation

Timber yield estimation for each age was taken from the growth curve of Chinese fir stands. The yield function estimated is expressed as:

$$\hat{Q}(t,m) = 5.85e^{-14.02/t^2 - 4678.67/tm - 7.31/t + 406.41/m}$$

Figure 3 shows changes in the amount of volume stock at various densities over time. Although similar in the early years, the growth curves clearly demonstrated that higher initial densities led to lower stock volumes after 10 years. The optimum rotation age calculated in Figure 3 relies entirely on biological information. The optimal biological rotations ranged from 10 to 13 years for Chinese fir plantations. The CAIs and MAIs were calculated using the yield function regression parameters substituted into equations 1 and 2. Figure 4 shows only the CAIs and MAIs of the highest and the lowest initial planting densities for clarity. It is obvious that higher initial planting densities shortened the biological rotation.

The optimal economic vs biological rotation

The volumes and values according to Faustmann's rule are much higher than those to MSY rule (Table 2). The initial planting density is important in determining the optimum rotation age under both rules. Lower initial planting densities increased the optimum rotation age. The optimal economic rotation ages of different initial planting densities under a discount rate of 0.05 occurred from 13 to 15 years, which were two to three years longer than the optimal biological rotations. Harvest volumes differed by over 20 m^3 ha^{-1}, so the profit at harvest age was higher

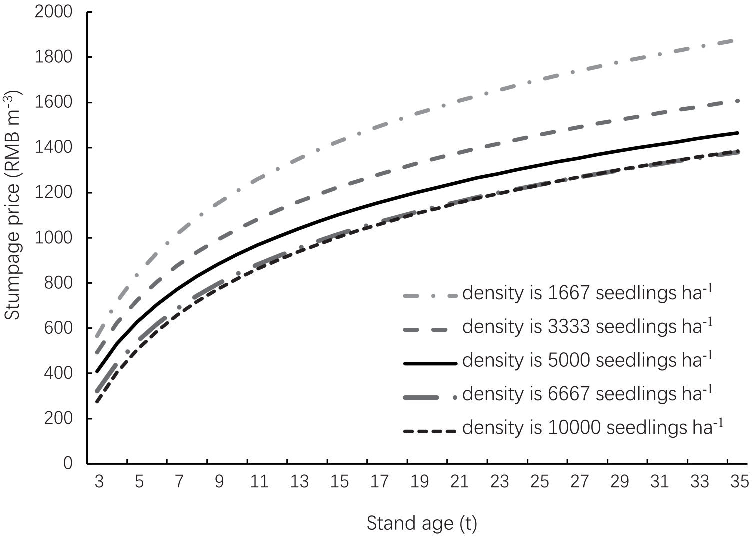

Figure 2 Average stumpage prices of stands under five initial planting densities predicted for Chinese fir

Table 2 Optimal rotation age, profit, harvest volume and benefit cost ratio with Faustmann and maximum sustainable yield (MSY) for Chinese fir

Initial planting density (seedling ha^{-1})	Faustmann Rule (r = 0.05)				MSY			
	Age (t**)	Profit (RMBha^{-1})	Volume (m^3 ha^{-1})	CR (%)	Age (t*)	Profit (RMBha^{-1})	Volume (m^3 ha^{-1})	CR (%)
1667	15	193583.85	227.19	7.79	13	157953.01	202.61	9.55
3333	14	172109.03	210.44	8.81	12	137693.92	186.93	11.12
5000	13	154566.16	198.27	9.87	11	120145.73	179.15	12.79
6667	13	154147.75	197.73	11.18	11	119204.33	173.24	12.86
10000	13	153730.61	197.19	11.31	10	100590.19	158.89	15.41

t* is the optimum biological rotation, t** is the optimum economic rotation, CR is the benefit cost ratio

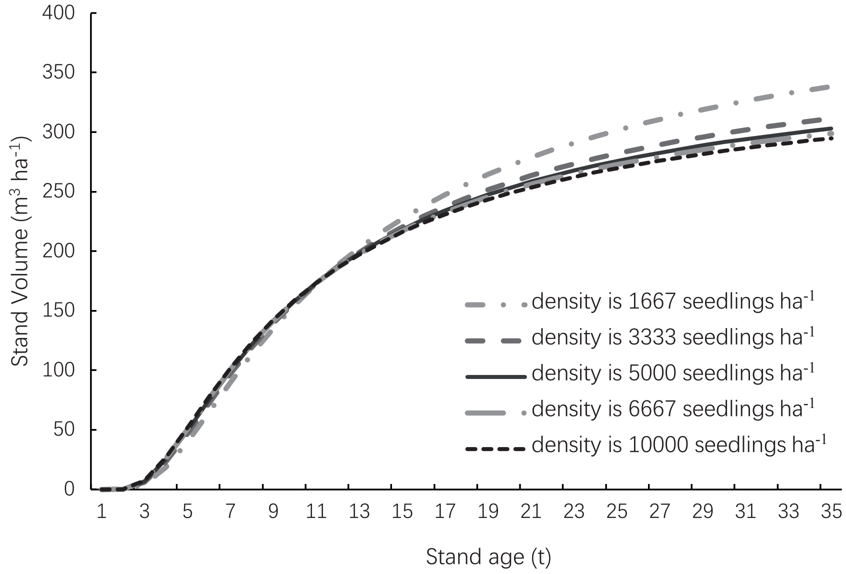

Figure 3 The amount of volume stock at various densities

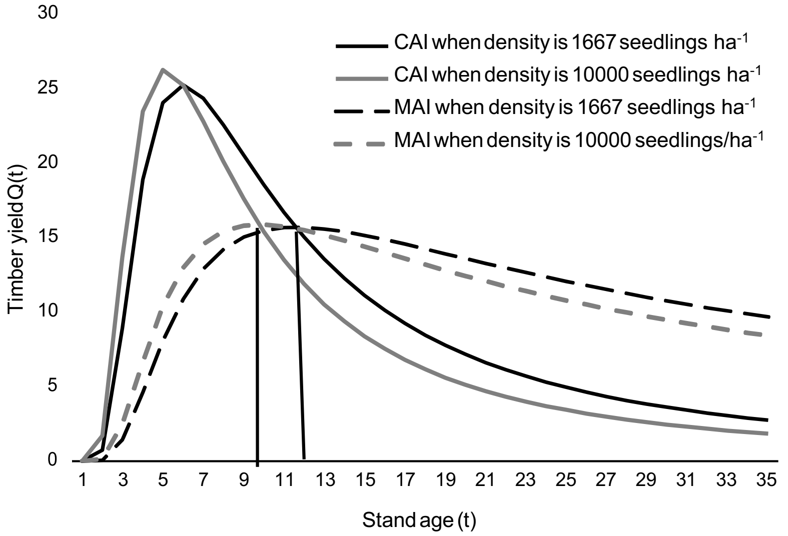

Figure 4 The relationship between CAI and MAI under different planting density

and CR were lower under the Faustmann criteria than MSY. These results have a close relationship with the optimum rotation age; the longer the optimum rotation age, the higher the volume and profit and the lower the ratio of c/pQ. Even if the rotation age is the same, a lower initial planting density leads to higher profits.

As shown in Table 3, it was found that the optimal biological rotation age may be shorter or longer than the Faustmann rotation age. The sensitivity analysis illustrated that the optimal economic rotation ages were always longer than the biologically optimal period when the discount rate was low and the differences between t^{**}s and t^{*}s became smaller with increasing discount rates. When the discount rate reached 0.07–0.09, t^{**} and t^{*} were equal when initial planting density was 1667 seedlings ha^{-1}. At the maximum density of 10000 seedlings ha^{-1}, t^{**} and t^{*} were equal when the discount rate reached 0.15.

DISSCUSSION

The results from empirical analysis based on Chinese fir survey data (Table 2 & 3, Figure 4) agreed with the results of the mathematical analysis (Figure 1, Table 1). The optimum rotations under both Faustmann and MSY criteria were sensitive to the initial planting density, stumpage price and discount rate. When the constant stumpage price assumption was relaxed, similar to Zhou (1999) and Brazee and Dwivedi (2015), it was found that forest managers would need to adjust their silvicultural decisions according to changing stumpage price and discount rates to satisfy both profit and sustainability.

The threshold of the annual incremental growth of Chinese fir, when timber should be harvested under the MSY criterion, occurred around 10–13 years. Similar results were found by previous studies (State Forestry Bureau of China 2007). This finding was expected as Chinese fir trees grow rapidly. However, the MSY threshold was unlikely to be the right time for forest managers to harvest. Chinese fir is usually cut as soon as the forest owners observe a reduction in timber growth. Related research, such as the technical standard of fast-growing and high-yielding Chinese fir timber plantation, sets the harvesting time for Chinese fir at 20–24 years (State Forestry Bureau of China 2007). Sheng et al. (1991) suggested 16–20 years as the harvesting period, and Chen et al. (2001) advised 21–40 years as the harvesting period. Several pieces of evidence were found that may contribute to these recommendations for later harvesting. First, there are three objective classes for silviculture, i.e., small, medium and large diameter timber. If the harvesting time is set to 10–13 years, there will be few large diameter trees in a stand. From an economic view, short rotation plantations may have higher equilibrium returns (Brukas et al. 2001). Producing more large diameter class timber may decrease the profit. However, nationally owned forestry operations will consider the strategic supply of timber besides the profit. Second, site conditions were taken into consideration in this research. The index of site conditions in the existing literature ranged from 12 to 18. In this paper, the data under the same site conditions had an index of 18. The higher the index of site condition, the shorter the harvest age, because the biological conditions determine the growth rate of trees in a stand (Sun et al. 2018). Site condition has a similar effect to initial planting density because stands with lower initial planting density use resources more efficiently. Therefore, the standing volume is higher at lower initial planting density, probably because of less competition (Hai & Egashira 2008). This finding is similar to that of Sun et al. (2011). As

Table 3 Differences between optimal economic rotation time (t^{**}) and the optimal biological rotation (t^{*}) for Chinese fir plantations with different initial planting densities and discount rates

Initial planting density (seedling ha^{-1})	Discount rate											
	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09	0.1	0.15	0.2
1667	5	4	4	2	2	1	0	0	0	-1	-2	-3
3333	5	4	3	3	2	1	1	0	0	0	-2	-3
5000	6	5	4	3	2	2	2	1	1	0	-1	-2
6667	6	5	4	3	2	2	1	1	1	0	-1	-2
10000	7	6	5	4	3	3	2	2	2	1	0	-1

shown in Table 2, lower initial planting densities led to shorter optimum rotations under both MSY and Faustmann criteria.

Differences were found between the optimal biological rotation and the optimal economic rotation. Table 2 confirmed that the optimal economic rotations were longer than the biologically optimum rotation ages when the discount rate was 0.05. Table 3 showed that for the lower discount rate, the optimum rotation age of MSY criterion was always shorter than the corresponding Faustmann rotation age. This result differed from the findings of a previous study which showed shorter economic rotations and longer biological rotations (Amacher et al. 2009). One of the reasons for this variation is that different types of prices were used. Olschewski and Benitez (2010) reported that the optimal financial rotation would be shorter than the MSY rotation when using a constant price. A higher price growth will extend the rotation so that the Faustmann rotation is longer than the MSY rotation. Nghiem (2014) stated that the use of the timber price function on age will cause the optimum rotation to be longer than using a constant timber price. In this study, a price that increased with the stand age was used, which is more realistic. Increasing prices led to longer rotations. Another reason has been proposed for relatively low discount rates which are well below the inverse value of t^* (Table 2). Binkley (1985) suggested that economic rotations would be greater than MSY rotations only when the discount rate was less than the inverse of the MSY rotation. Thus, for slow growing species, such as many temperate hardwoods where the MSY rotation may be almost 100 years, the economic rotation will be shorter than the MSY rotation. However, for fast growing species, such as tropical forest plantations which reach the culmination of MAI in less than 20 years, the economic rotation will be longer than the MSY rotation.

One of our most interesting findings was the conditions that could lead the rotations of MSY to equal the Faustmann rotations. Many previous researchers believed that MSY harvest ages became a special case when a zero rate of interest was used in the Faustmann formula (Chang 2020). Binkley (1985) showed that this conclusion was valid only if regeneration and management costs were ignored. Reed (1986) also reported that if we assume no substitutability between land and capital, the maximisation of the economic return under an area restriction provides a solution where the optimum rotation age is independent of the interest rate, and equal to the MSY rotation age. By comparing rotations between MSY and the Faustmann rules in this study, new light had been shed on the relationship between the discount rate, the harvesting age and the increase rate of stumpage price. We found that when the discount rate equaled the inverse of the MSY rotation age plus the increment of stumpage price, the Faustmann rotation age approached the MSY rotation age. It should be noted that the c/pQ ratio should be low enough that the production process is profitable. In the example of Chinese fir, we found that when the discount rate was as high as 0.15, the Faustmann rotation equaled the MSY rotation age of 10 years old when the initial planting density was 10,000 plantings per ha. The ratio of the regeneration costs to the value of timber equaled approximately 15%, which is lower than $e^{-(1+t\frac{P_t}{p})} = 22\%$, to keep the planting management profitable.

CONCLUSION

This study searched for the decisive conditions for optimum rotation in the context of both MSY and Faustmann criteria. The assumption of the Faustmann model that the stumpage price is a function of stand age was relaxed, and the optimal economic rotation was analysed with its dependence on the natural conditions of growth and effects of discount rates. This may help pinpoint some of the likely differences between the classic Faustmann rotation and MSY rotation for even-aged plantation forest. The more realistic assumption resulted in new findings. First, when the discount rate equaled the inverse of the MSY rotation age plus the increment of stumpage price, the Faustmann rotation age approached the MSY rotation age. At this point, the c/pQ ratio will be low enough (less than $e^{-(1+t\frac{P_t}{p})}$) that the production process is profitable. Second, it was found that equalising rotations under MSY and Faustmann frame were particularly sensitive to the choice of initial planting densities; the lower the initial planting density, the longer the optimal biological rotation. A lower discount rate will make the rotations under MSY and Faustmann frame coincide. This information will provide insights into whether silvicultural activities are a sensible use of capital.

From the analysis, it was also found that forest managers can set the initial planting density, and the national policy makers can adjust the interest rate of forest investment to improve forest plantation management. For the policy maker, the interest rate of forest loans should be set based on the increment of stumpage price which should be higher for fast-growing tree species than slow-growing tree species. For forest managers, the planting decision should take initial planting density into account and try to decrease the management costs which should not be high enough to cause the economically optimum rotation to fall behind the biologically optimum rotation. Economic and sustainability objectives will be achieved when the density and management costs are coordinated. If profit is to be considered, a lower initial planting density is recommended to manage Chinese fir plantations.

One obvious caveat in this work is that thinning was not considered, which is an important factor for the growth function and the Faustmann rotation. One reason was that the yield model used was originally estimated based on measurements in stands that had not undergone thinning. The growth reaction to thinning may vary, affecting the optimum sustainability of the solutions. Another reason is that thinning requires the trees to be removed which may result in increased costs and additional logging damage for some stands. Finally, the inclusion of thinning would increase the model's complexity which might not yield meaningful results. However, thinning could improve timber quality and the merchantable stand volume, and therefore, increase operating incomes. Future analyses that include thinning would provide a more complete understanding of forest management.

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REFERENCES

Amacher GS, Ollikainen M & Koskela E. 2009. *Economics of Forest Resource.* MIT Press, Cambridge MA. doi: 10.1093/erae/jbq028.

Battaglia M & Sands PJ. 1998. Process-based forest productivity models and their application in forest management. *Forest Ecology and Management* 102: 13–32. doi: 10.1016/S0378-1127(97)00112-6.

Bettinger P & Tang M. 2015. Tree-level optimization for structure-based forest management based on the species mingling index. *Forests* 6: 1121–1144. doi: 10.3390/f6041121.

Binkley CS. 1985. *Long Run Timber Supply: Price Elasticity, Inventory Elasticity, and The Capital-Output Ratio.* IIASA Working Paper. IIASA, Laxenburg, Austria. doi: 10.1111/j.1939-7445.1993.tb00145.x.

Binkley CS. 1987. When is the optimal economic rotation age longer than the rotation age of maximum sustained yield? *Journal of Environmental Economics and Management* 14: 152–158. doi: 10.1016/0095-0696(87)90013-1.

Brazee RJ & Dwivedi P. 2015. Optimal forest rotation with multiple product classes. *Forest Science* 61: 458–465. doi: 10.5849/forsci.13-207.

Brazee R. 2018. Impacts of declining discount rates on optimal harvest age and land expectation values. *Journal of Forest Economics* 31: 27–38. doi: 10.1016/j.jfe.2017.06.002.

Brukas V, Thorsen BJ, Helles F & Tarp P. 2001. Discount rate and harvest policy: implications for Baltic forestry. *Forest Policy and Economics* 2: 143–156. doi: 10.1016/S1389-9341(01)00050-8.

Cairns RD. 2017. Faustmann's formulas for forests. *Natural Resource Modelling* 30: 52–73. doi: 10.1111/nrm.12105.

Chang SJ. 1984. A simple production function model for variable density growth and yield modelling. *Canadian Journal of Forest Research* 14: 783–788. doi: 10.1139/x84-139.

Chang SJ. 2020. Twenty-one years after the publication of the generalized Faustmann formula. *Forest Policy and Economics* 118: 102238. doi: 10.1016/j.forpol.2020.102238.

Chen PL, Liu J & Zheng DX. 2001. The confirmation of harvesting period and the analysis of economic effect on the productive and high-quality Chinese fir plantation. *Scientia Silvae Sinicae* (in Chinese) 37: 47–51.

Dykstra DP. 1986. The influence of logging costs on optimum rotation age for Site I *Pinus patula* in Tanzania. Systems Analysis in Forestry and Forest Industries. *North-Holland Elsevier Science Publishers* 1: 297–310.

Gaffney MM. 1960. *Concepts of Financial Maturity of Timber and Other Assets.* A.E. In-formation Series No. 62. North Carolina State College Raleigh, North Carolina.

Hai NQ & Egashira K. 2008. Clay mineralogy of ferralitic soils derived from igneous rocks in Vietnam. *Clay Science* 13: 189–197. doi: 10.11362/jcssjclayscience1960.13.189.

Hartman R. 1976. Harvesting decision when a standing forest has value. *Economic Inquiry* 14: 52–58. doi: 10.1111/j.1465-7295.1976.tb00377.x.

Ivanova N. 2017. *Research Methods of Timber-Yielding Plants. Biology, Productivity and Bioenergy of Timber-Yielding Plants.* Springer Briefs in Plant Science, Springer, Cham, Heidelberg. doi: 10.1007/978-3-319-61798-5_3.

Jacobsen JB, Jensen F & Thorsen BJ. 2018. Forest value and optimum rotations in continuous cover forestry.

Environmental and Resource Economics 69: 713–732. doi: 10.1007/s10640-016-0098-z.

Kaya A, Bettinger P, Boston K et al. 2016. Optimization in forest management. *Current Forestry Reports* 2: 1–17. doi: 10.1007/s40725-016-0027-y.

Liu JF & Tong SZ. 1980. Studies on the stand density control diagram for *Cunninghamia Lanceolata*. *Scientia Silvae Sinicae* 16: 241–251.

Lobovikov M & Nga TTT. 2019. Economic criteria of forest management. IOP Conference Series. *Earth and Environmental Science* 226: 012033. doi: 10.1088/1755-1315/226/1/012033.

Luckert MK & Williamson T. 2005. Should sustained yield be part of sustainable forest management? *Canadian Journal of Forest Research* 35: 356–364. doi: 10.1139/x04-172.

Mills WL & Jackson DH. 1980. The microeconomics of the timber industry. *Westview Replica Editions* 63: 405. doi: 10.2307/1239598.

Nautiyal JC & Couto L. 1982. The use of production-function analysis in forest management: eucalypts in Brazil, a case study. *Canadian Journal of Forest Research* 12: 452–458. doi: 10.1139/x82-070.

Newman DH, Gilbert CB & Hyde WF. 1985. The optimal forest rotation with evolving prices. *Land Economics* 61: 347–353. doi: 10.2307/3146152.

Nghiem N. 2014. Optimum rotation age for carbon sequestration and bio-diversity conservation in Vietnam. *Forest Policy and Economics* 38: 56–64. doi: 10.1016/j.forpol.2013.04. 001.

Olschewski R & Benitez PC. 2010. Optimizing joint production of timber and carbon sequestration of afforestation projects. *Journal of Forest Economics* 16: 1–10. doi: 10.1016/j.jfe.2009.03.002.

Price C. 2017. Optimum rotation with differently-discounted benefit streams. *Journal of Forest Economics* 26: 1–8. doi: 10.1016/j.jfe.2016.10.001.

Reed WJ. 1986. Optimal harvesting models in forest management a survey. *Natural Resource Modelling* 1: 55–79. doi: 10.1111/j.1939-7445.1986.tb00 003.x.

Reed WJ. 1993. The decision to conserve or harvest old-growth forest. *Ecological Economics* 8: 45–69. doi: 10.1016/0921-8009(93)90030-A.

Sheng WT, Hui GY & Luo YW. 1991. A study on felling age of Chinese fir plantation. *Forest Research* (in Chinese) 2: 113–121.

State Forestry Bureau Of China. 2007. *Technical Standard of Fast-Growing and High-Yielding Chinese Fir Timber Plantation.* No. LY/T 1384-2007. Standard Publishing House, Beijing. doi: 10.1139/cjfr-2016-0310.

Sun HG, Zhang JG, Duan AG & He CY. 2011 Estimation of the self-thinning boundary line within even-aged Chinese fir (*Cunninghamia lanceolata* (Lamb.) Hook.) stands: onset of self-thinning. *Forest Ecology and Management* 261: 1010–1015. doi: 10.1016/j. foreco. 2010.12.019.

Sun HG, Diao SF, Liu R et al. 2018. Relationship between size inequality and stand productivity is modified by self-thinning, age, site and planting density in *Sassafras tzumu* plantations in central China. *Forest Ecology and Management* 422: 199–206. doi: 10.1016/j. foreco.2018.02.003.

Tahvonen O, Salo S & Kuuluvainen J. 2001. Optimal forest rotation and land values under a borrowing constraint. *Journal of Economic Dynamics and Control* 25: 1595–1627. doi: 10.1016/S0165-1889(99)00065-2.

Washburn CL & Binkley CS. 1990. On the use of period-average stumpage prices to estimate forest asset pricing models. *Land Economics* 66: 379–393. doi: 10.2307/3146620.

Zhou W. 1999. Optimal method and optimal intensity in reforestation. PhD thesis. Swedish University of Agricultural Sciences, Umea, Sweden.